Jackson

VOL. I.

ELECTRO-MAGNETISM

AND THE

CONSTRUCTION OF CONTINUOUS CURRENT DYNAMOS.

A TEXT-BOOK

ON

ELECTRO-MAGNETISM

AND THE

CONSTRUCTION OF DYNAMOS.

VOL. I.

BY

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PREFACE.

THIS book has been developed from the lectures which the writer has presented to his classes in dynamo construction. No special originality is claimed for the matter presented except in certain details; but it is hoped that the logical arrangement, and the omission of all unnecessary details, will make the book of so convenient a form as to lead to its general endorsement as a satisfactory text and reference book. Since the primary object in publishing the book is to supply a satisfactory text-book to be used as the basis of instruction to college classes, certain of the fundamental principles may possibly be more fully presented than would be desirable in a mere reference book. With the same point in view, no cuts or descriptions of typical dynamos are placed in the pages. The student is expected to gain a familiarity with the different commercial machines in his work in the college laboratories, and to add to the familiarity by inspection tours made under proper direction.

The present volume does not touch upon alternating current machinery nor series arc lighting machinery,

but in the near future a second volume will be issued treating of these.

The time has not yet come when a proper history of the development of the American dynamo can be written; therefore this book does not touch upon the historical side of the subject. As most of the literature of the subject is of transatlantic origin, comparatively few references can be made to American writers. On the other hand, the book treats the subject from the American standpoint and represents present American practice.

It is impossible to mention all the sources from which information has been drawn, but the writer is specially indebted to Professor Merritt's *Notes on Dynamo Electric Machinery;* Kapp's *Dynamos, Alternators, and Transformers;* Thompson's *Lectures on the Electro-magnet;* Ewing's *Magnetic Induction in Iron and Other Metals;* papers by Dr. John Hopkinson and other well-known writers; and the designers of many of the best American machines. Acknowledgment is also due Professor J. P. Jackson of the Pennsylvania State College for his kindness in reading proof.

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CONTENTS.

ELECTRO-MAGNETISM.

CHAPTER I.

PRIMARY DEFINITIONS AND EVALUATIONS.

A Magnetic Field is any space within which a magnet pole is acted upon by magnetic forces.

The Strength of Field, at any point in a magnetic field, is measured by the force in dynes exerted upon a unit magnet pole placed at the point. Strength of field is usually represented by the capital letter H. Since force is the rate of change of potential, H evidently represents the difference of magnetic potential per unit length. Difference of magnetic potential is frequently called **Magneto-Motive Force,** or **Magnetic Pressure** (analogies, the terms electro-motive force, or electric pressure, which are used to designate difference of electric potential).

A Unit Magnet Pole is one which acts with a force of one dyne upon an equal pole placed at a distance of one centimeter.

A Magnetic Field of Unit Strength is one within which a unit pole experiences a force of one dyne. The total strength of a magnetic field may be conveniently conceived as the resultant of many unit forces,

each of which has its individual line of action. These lines of action are called **Lines of Force,** and the field of unit strength may be defined as having one line of force per square centimeter. Since a unit pole exerts a force of one dyne upon an equal pole placed at one centimeter distance, there is unit field at the latter point. A sphere of one centimeter radius has a surface of 4π square centimeters, whence, from the second definition of unit field, 4π lines of force must emanate from a unit pole.

The conception of lines of force is very useful and should be thoroughly grasped; it must always be remembered however that such lines have not a material existence, but are merely hypothetical.

It is evident that a line of force must always join two points of different potentials, and that two lines of force can never intersect. The positive direction along a line of force is from high potential to low potential, in which direction a free north pole tends to move.

When the lines of force in a magnetic field are parallel and of equal number per square centimeter, a magnet pole will experience the same force at all points of the field, and the field is said to be **Uniform.**

If, as shown above, 4π lines of force emanate from a unit pole, it is evident that $4\pi m$ lines of force must emanate from a pole of strength m. If, instead of being a mathematical point, the magnet pole of strength m be a sphere of one centimeter radius, m lines of force must emerge per square centimeter of surface. If the radius be r, the number of lines of force per square

centimeter is $\frac{4\pi m}{4\pi r^2}=\frac{m}{r^2}$. A free spherical magnet pole is a physical impossibility, as each pole must be attached by magnetic material to another pole of equal strength but of opposite sign, and the lines of force which emerge from one pole end in the other. However, the definitions given may be applied equally to the physical pole. Suppose that the polar end of an ordinary magnet be flat, with an area of A square centimeters, and pole strength m; then the total number of lines of force emerging from the pole is $4\pi m$, and the number of lines per square centimeter of pole surface is $\frac{4\pi m}{A}$.

The ratio $\frac{m}{A}$ is called **Intensity of Magnetization,** and is usually represented by the capital letter I. The magnetic moment of a magnet is ml (where l is the length of the magnet) and the volume of the magnet is Al. It is therefore evident that

$$\frac{m}{A}=\frac{\text{Magnetic Moment}}{\text{Volume}},$$

and the intensity of magnetization can be defined as the magnetic moment per unit volume. This definition served a very useful purpose in the hands of the early mathematicians, but a more useful definition for practical service is, strength of pole per unit of polar area. From the definition of I it is seen that the strength of field very close to a magnet pole is $4\pi I, \left(=\frac{4\pi m}{A}\right)$, and the total number of lines of force emerging from the pole is $4\pi IA$, $(=4\pi m)$.

If a piece of magnetic material be placed in a magnetic field of strength H, it increases the number of lines of force at the point by acquiring an intensity of magnetization I, which depends upon the magnetic quality of the material. The ratio $\frac{I}{H}$ in any case is called the **Susceptibility** of the material and is represented by Greek κ. $\kappa = \frac{I}{H}$. The susceptibility of iron, nickel, cobalt, and possibly other less known metals, is very large, and is a function of I. For air and nearly all other materials, whether metal or otherwise, it is zero or a very small negative fraction. With the exception of the magnetic metals named, it can be practically considered as zero for all materials and for a vacuum.

The total number of lines of force per square centimeter, which are induced in and emanate from the poles of a piece of magnetic metal (iron for instance), when placed in a magnetic field, equals $H+4\pi I$. The first term is the strength of the field (lines of force per square centimeter) before the iron was placed within it, and $4\pi I$ is the increase close to the poles due to the presence of the metal. The introduction of iron in the field sometimes changes the numerical value of H, but the general statement remains unaltered. Each one of the $H+4\pi I$ lines of force which emerge from or come to the poles can be considered as passing through the iron from south pole to north pole, thus making closed curves with the lines of force which pass from north pole to south pole outside of the iron. Within the iron these lines are often called by physicists **Lines of Magnetization** or **Lines of Induction**; but the distinction is not necessary or useful

in the practical applications. We will therefore consider a line of force as a closed curve which passes from the north to the south pole outside of a magnet, and continues from the south pole to the north pole within the material of the magnet, as shown in Fig. 1.

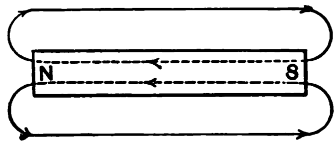

Fig. 1.

From what has preceded, it is evident that the total number of lines of force per square centimeter within a magnet is $H+4\pi I$. This is usually called the **Induction**, and is designated by the capital letter B. Hence, $B=H+4\pi I$. $\int BdA$ is usually called the **Total Magnetization**, or **Total Induction**, and is frequently designated by the capital letter N. When B is uniform over the area A, we have

$$N=BA=HA+4\pi IA=HA+4\pi m.$$

Since $I=\kappa H$, we have

$$B=H+4\pi\kappa H=H(1+4\pi\kappa).$$

Since κ is a function of I for all magnetic metals, as already stated, therefore $1+4\pi\kappa$ is a function of B. $1+4\pi\kappa$ is called the **Magnetic Permeability*** of the metal, and is designated by Greek μ. Thus,

$$\mu=1+4\pi\kappa, \text{ whence } B=\mu H.$$

* The word permeability was first suggested by Lord Kelvin to represent the mathematical conception of what may be called specific magnetic conductivity, which is in reality the ratio of B to H.

For air, and other non-magnetic materials, κ is practically equal to zero, and therefore $\mu = 1$. Permeability is equivalent to specific "magnetic conductivity"; that is, the permeability of a metal is the same as the magnetic conductivity of a piece one centimeter in length and one square centimeter in cross-section. Hence, if the value of μ, the summation of H, $(=\int H dl)$, and the length, l, of any magnetic circuit making a complete or nearly complete ring, are known, the induction is given by the formula

$$B = \frac{\mu}{l} \int H dl.$$

$\int H dl$ is the total magnetizing force, or magnetic pressure, in the circuit, and is often designated by the capital letter M. When B is uniform over an area, A, we have

$$N = BA = \frac{A\mu}{l} M.$$

We have here a fundamental law of the magnetic circuit that for practical purposes is entirely analagous to Ohm's law for electric circuits, which is usually represented by the formula $C = \frac{E}{R}$. If R be replaced in the latter expression by $\frac{1}{K}$, the reciprocal of conductivity, we have $C = KE$. Again, if k be used to represent specific electrical conductivity, the expression becomes

$$C = \frac{Ak}{l} E,$$

which is exactly similar to the expression for N. As a matter of convenience, the total magnetizing force M may be called **Magneto-Motive Force**, or **Magnetic Pressure**

(analogies, electro-motive force, or electric pressure).* The total magnetization N may also be called **Magnetic Flux,** or **Flow** (analogy, electric flux, or current). The expression of Ohm's law for the electric circuit, and its counterpart for the magnetic circuit, thus becomes similar to the expression for the flow of water, gas, etc., through pipes, where the flow in cubic feet per second equals the pressure divided by the resistance to motion, $Q=\frac{P}{F}$. The resistance in this case is usually due to skin friction, and is a function of the velocity of flow; hence it is not strictly analogous to electric or magnetic resistance, but this does not vitiate the similarity of the general expressions. Another analogy is the stretch of material when under strain. The stretch is equal to the applied stress divided by the elastic force resisting deformation. From the analogies it may be seen that the law of Ohm is a special statement of the results of ordinary observation, and it may be generalized thus: a result is equal to the effort put forth divided by the opposing resistance or opposition.

It is advantageous in many cases to use terms signifying magnetic resistance instead of magnetic conductivity, and it is convenient to designate the reciprocal of μ by Greek ρ. The expression for N then becomes $N=\frac{A}{l\rho}M$, where ρ represents specific magnetic resistance, and $\frac{l\rho}{A}$ is the actual magnetic resistance of a body l in length and A in cross-section. Magnetic resistance is usually called **Reluctance,** and may be designated by P.

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* Compare page 1.

The ordinary electric and magnetic circuits differ materially in two properties. Thus, a conductor carrying an electric current can be readily insulated by various materials, including dry air. On the other hand, as already stated, the magnetic permeability of all non-magnetic materials is practically unity, and, therefore, thorough magnetic insulation cannot be effected. This leads to the analogy of an electric battery immersed in sea-water, which was first pointed out by Faraday. It is evident that when a battery with attached wires is immersed in sea-water, or other poorly conducting liquid, much of the current will follow the wires through their resistance *R*, while some of the current will pass through the liquid between the battery poles, as shown by the dotted lines in Fig. 2. The currents passing by the two paths are inversely as the respective resistances.

The case of the magnetic circuit is represented by Fig 3, which is an iron ring with a narrow break at *R*, and a magnetizing coil at *C*. Many of the lines of force set up in the ring follow the iron all the way and jump the break at *R*,

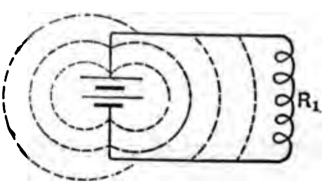


Fig. 2.

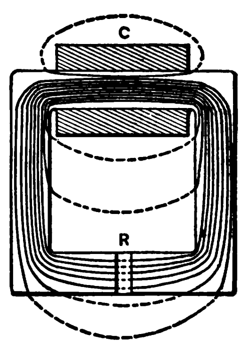


Fig. 3.

while others jump across between the magnet limbs or pass directly around the coil. As in the electric

circuit, the total induction in each path is inversely proportional to the reluctance of the path. Thus $N_r = \frac{M}{P_r}$ and $N_a = \frac{M}{P_a}$, where N_r, N_a, P_r and P_a are the total inductions and reluctances through the gap and air respectively.

The second essential difference between electric and magnetic circuits is the variation of μ with B for magnetic metals, while the electric resistance of metals is unchanged by the flow of a current, provided the temperature is not changed. As already stated, μ is practically constant for all non-magnetic materials, while its variation in magnetic metals is fairly analogous to the change of electric resistance in a circuit which carries an electric current sufficiently large to heat it. In this case the resistance is a more or less complex function of the current.

The following rather unsatisfactory representation of the magnetic field is quoted from *Practical Electrical Engineering:* "Imagine a magnet represented by a tube, in the centre of which is a screw pump, and let the tube be immersed in water whilst the pump is rotated. The water will issue at one end, flow in curved stream lines and varying velocities about the tube, and enter it again at the other end. The unit exploring pole we replace by a disc of unit surface, which we place in various positions within the space surrounding the tube, and thus measure the force of the stream at any point. This analogy is imperfect, because the force exerted by the water varies not as the velocity, but as the square of the velocity. Assuming, however, that the

former be the case, then such a model can in a somewhat crude fashion be made to represent the magnetic field. We may think of the lines of force, not as a definite number of fixed lines threading through the space which separates the two poles of the magnet, but as the stream lines of a kind of magnetic fluid circulating through this space. Near the poles of the magnet the stream is contracted, and the velocity is therefore great. In these places the force of impact of the magnetic fluid upon the exploring pole is a maximum, whilst farther away from the poles where the velocity, consequent upon the expansion of the stream, is less, the force of impact is also less. In this manner can be explained the variation of magnetic force as we move the exploring pole into different parts of the field, and the fact that a magnetic field taken by itself represents a definite amount of stored-up energy."

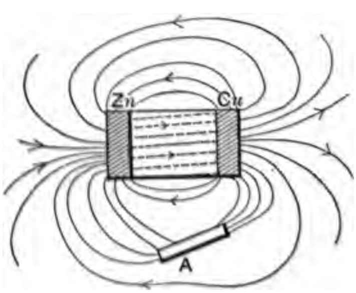


Fig. 4.

Returning to the analogue of a voltaic cell in salt water, the idea of stream lines may be carried farther. Let *Zn, Cu,* in the figure, represent a cell which has its circuit closed only by the surrounding water; then the current flowing between the poles of the cell will be distributed in stream lines through the water. These electric stream lines, or lines of flow, can be represented, as in Fig. 4, by lines drawn so that they show the direction of flow (or force)

at every point, and by their number indicate the density of the flow or current. The lines thus drawn are similar in form to the liquid stream lines of the mechanical magnet explained above. They are also exactly similar to the lines of force due to a physical magnet of the same size and shape as the *Zn-Cu* element. If a good conductor, such as a copper rod, be placed at some point, as *A*, the current will flow through it by preference on account of its higher conductivity, and the stream lines will be deflected. In the same way, a piece of iron in a magnetic field will apparently gather to itself the lines of force, on account of the greater conductivity of the path through it.

The close analogy between electric and magnetic circuits is quite useful in understanding the phenomena of the latter, but, as already explained, it is essential that the fundamental differences be carefully remembered. It is also necessary to remember that a magnetic flux cannot be actually treated as a flow of something material, as is usual in the case of the electric current, because there is nothing in the magnetic circuit analogous to the loss of energy caused by friction in a mechanical model, or by the C^2R loss when an electric current flows through a resistance. A magnetic field once set up requires no energy to maintain it.

It is proved in Thompson's *Electricity and Magnetism*, Art. 318, etc., that the potential at any point due to a closed electric circuit or magnetic shell, is equal numerically to the product of current strength, turns of the circuit, and the solid angle subtended by the circuit at the point. That is, $V = ncw$, V being potential, c cur

rent, n turns, and w solid angle. If the point be taken very close to the plane of the circuit, as in Fig. 5, w becomes 2π on one side and -2π on the other side of the plane. Hence the potential changes by $4\pi nc$ in passing from one side of the plane to the other, and therefore $4\pi nc$ ergs of work must be done in moving a unit pole around the circuit from one point to the other. Thus $W=4\pi nc$, where W=work in ergs. If c be used to represent current in amperes instead of in absolute units, the work becomes $W=\frac{4\pi nc}{10}$, since the ampere is $\frac{1}{10}$ the absolute unit of current. As work is equal to force × distance, there follows, $W=\int Hdl$, and hence $\frac{4\pi nc}{10}=\int Hdl$.

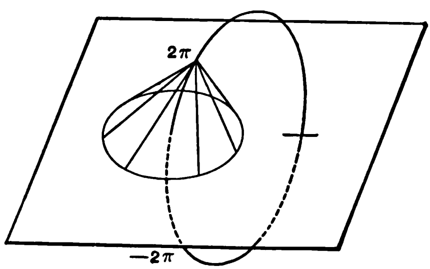


Fig. 5.

A cylindrical solenoid composed of a uniform layer of wire may be considered as made up of many exceedingly thin plane circuits, each acting as a magnetic shell. The force exerted upon a unit pole at a point, such as P in Fig. 6, within the solenoid and between two of the shells, must be the resultant effect due to one side of each of four shells,— the two immediately adjacent to the point, and the two end shells. All other magnetic effects are neutralized by the abutting faces of the various shells. By Thompson's *Electricity and Magnetism*, Art. 253, it is shown that the force at any point due to an attracting plate is $w\sigma$, where w

is the solid angle which the plate subtends at the point, and σ is the density of the attracting matter per unit area of the plate. For a plate with an electric charge, σ signifies the charge per unit area, and in the case of one side of a magnetic shell, σ is the magnetic intensity, equal to $\frac{nc}{10l}$.* Since $w=2\pi$ for the adjacent shells, one of these attracts the unit pole with a force which is parallel to the axis of the solenoid, and equal to $\frac{2\pi nc}{10l}$ dynes, while the other shell repels the pole with an equal

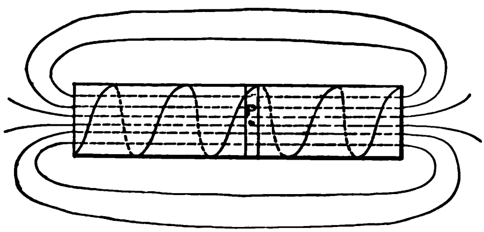


Fig. 6.

force. These two forces are in the same direction, hence the total force on the unit pole due to the adjacent shell faces is $\frac{4\pi nc}{10l}$ dynes. The end shells each exert a force on the unit pole respectively of $\frac{w_1 nc}{10l}$ and $\frac{w_2 nc}{10l}$, one being attraction and the other repulsion. The forces from the end shells are concordant, and their joint effect is therefore $\frac{w_1 nc}{10l}+\frac{w_2 nc}{10l}$, which is opposite in direction to the force exerted by the adjacent shells. The total force exerted on a unit pole within the solenoid is thus

* Where n is the total number of turns of wire on the solenoid, l is its length, and $\frac{n}{l}$ can therefore be taken as the number of turns on each shell.

$$H=\frac{4\pi nc}{10l}-\left[\frac{w_1 nc}{10l}+\frac{w_2 nc}{10l}\right] \text{ dynes.}$$

If the solenoid has an indefinitely great length on either side of the point, w_1 and w_2 are zero, and the force becomes $H=\frac{4\pi nc}{10l}$ dynes, and it is uniform throughout the solenoid. Whence, the work required to carry a unit pole around a path linking the solenoid, is

$$W=\int Hdl=Hl=\frac{4\pi nc}{10} \text{ ergs.}$$

If the solenoid be of finite length, there is no method of exactly measuring the solid angles w_1 and w_2 unless the point be taken on the axis. In the latter case, if a_1 and a_2 be the angles subtended at the point by the radii of the end faces, we have by geometry

$$w_1=2\pi(1-\cos a_1)$$

and $$w_2=2\pi(1-\cos a_2),$$

whence $$H=\frac{2\pi nc}{10l}(\cos a_1+\cos a_2) \text{ dynes,}$$

and H is not uniform. The latter fact makes it impossible to directly compute the work, $(=\int Hdl)$, required to carry a unit pole around a path linking the solenoid, but an approximation of considerable exactness can be made in many cases. When the point is taken at the centre of the solenoid, $\cos a_1+\cos a_2$ becomes

$$2\cos a, \text{ and } \cos a=\frac{l}{D},$$

l being the length of the solenoid and D the diagonal.

In any solenoid, H becomes practically uniform and equal to $H=\frac{4\pi nc}{10l}$ dynes, with an error not exceeding about 1% at three diameters distant from either end. Hence the work required to carry a unit pole around a path linking any solenoid of greater length than six diameters is approximately $W=Hl=\frac{4\pi nc}{10}$ ergs, and H at the middle cross-section is $\frac{4\pi nc}{10l}$ dynes.

In the case of a solenoid bent around in the shape of a ring, the end faces neutralize each other, and $H=\frac{4\pi nc}{10l}$ dynes, where l is the length of a circle concentric with the ring, and in which the reference point is taken. Since l is different for circles of different radii within the ring, it is evident that H will have different numerical values for each point upon any radial line. As the value of l is the least at the inner edge of the ring, H is there the greatest. The work done in carrying a unit pole through the solenoid is, as before, $W=Hl=\frac{4\pi nc}{10}$ ergs, and is therefore the same for all circles.

It is evident that only the end faces of a solenoid affect the potential at any outside point. From the preceding proofs, it is readily seen that the force upon a unit pole outside a solenoid is $\frac{w_1nc}{10l}+\frac{w_2nc}{10l}$ dynes. In the case of a ring, there are no end faces, and no outside force is exerted.

In each case, it is to be noted that $\frac{n}{l}$ is equal to the number of turns of wire on the solenoid per centimet

of length, and it might be written $\frac{n}{l}=t$, whence for long solenoids

$$H=\frac{4\pi tc}{10} \text{ dynes.}$$

Returning to the closed electric circuit in one plane, as given on page 11, the potential V, at any point outside of the shell, is $\frac{ncw}{10}$. If the circuit be in the form of a circle with radius r, and the point be on the axis at a distance x from the plane of the circuit, as in Fig. 7, w becomes

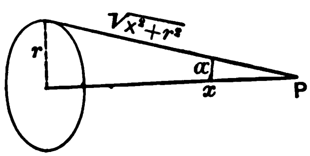


Fig. 7.

$$2\pi(1-\cos a)=2\pi\left(1-\frac{x}{\sqrt{r^2+x^2}}\right),$$

whence $$V=\frac{2\pi nc}{10}\left(1-\frac{x}{\sqrt{r^2+x^2}}\right).$$

Since force is the rate of change of potential,

$$H=-\frac{dv}{dx}{}^{*}=\frac{2\pi r^2 nc}{10(x^2+r^2)^{\frac{3}{2}}} \text{ dynes,}$$

the current being given in amperes.

When the point is at the centre of the circle,

$$x=0, \text{ and } H=\frac{2\pi nc}{10r}.$$

When such a coil, with a magnetic needle at its centre, is placed with its plane parallel to the magnetic meridian, it becomes a single-coil tangent galvanometer.

* The negative sign is used here because force is exerted from high potential to low potential.

The couple exerted by the coil upon the needle is evidently $\frac{2\pi nc}{10r} lm \cos\theta$ dynes, where lm is the magnetic moment of the needle, and θ is the angle of deflection measured from the plane of the coil. The earth's magnetism exerts on the needle a couple of $H_e lm \sin\theta$ dynes, which balances that of the coil when the needle is at rest. Hence

$$\frac{2\pi nc}{10r} lm \cos\theta = H_e lm \sin\theta,$$

and therefore

$$c = \frac{10 H_e r}{2\pi n} \tan\theta,$$

where c is in amperes. $\frac{2\pi n}{10r}$ is usually called the principal constant of the galvanometer, and is often designated by G, when the formula is written $c = (H_e/G)\tan\theta$. A more reliable form of the tangent galvanometer is made with two parallel coils of equal diameter, separated a distance $2x$, which is about equal to the radius. The needle is centred upon the common axis midway between the planes of the coils. In this case the force exerted by the two coils upon the deflected needle is

$$\frac{4\pi r^2 nc}{10(x^2+r^2)^{\frac{3}{2}}} lm \cos\theta \text{ dynes.}$$

The earth exerts a force, as before, of $H_e lm \sin\theta$ dynes; whence

$$\frac{4\pi r^2 nc}{10(x^2+r^2)^{\frac{3}{2}}} lm \cos\theta = H_e lm \sin\theta$$

and

$$c = \frac{10 H_e (x^2+r^2)^{\frac{3}{2}}}{4\pi n r^2} \tan\theta.$$

When x equals exactly $\frac{1}{2}r$, this becomes

$$c = \frac{50\sqrt{5}\,H_e r}{32\pi n}\tan\theta.$$

These formulas are ordinarily used when currents are measured by the tangent galvanometer, but when extremely refined measurements are to be effected, certain corrections are necessary on account of the finite dimensions of the needle, coils, etc. These, however, need not be considered here.

The previous discussion shows that each face of one of the elementary slices or shells of a long solenoid is equivalent to $\frac{nc}{10\,l}A$ unit poles, and that each unit pole in a face has exerted upon it a force of $2\pi\frac{nc}{10\,l}$ dynes by the shell faces upon each side of it. The total attractive force exerted upon a shell face is, therefore,

$$2\pi\frac{nc}{10\,l}\times\frac{nc}{10\,l}A = 2\pi\left(\frac{nc}{10\,l}\right)^2 A \text{ dynes.}$$

But $$2\pi\left(\frac{nc}{10\,l}\right)^2 A = \frac{H^2 A}{8\pi}.$$

Within a solenoid which has no magnetic core, $H = B$, hence the force between opposing faces is $\frac{B^2 A}{8\pi}$ dynes. This force acts as though there were a tension along the lines of force equal to $\frac{B^2}{8\pi}$ dynes per square centimeter. Where B is not uniform over the area A, the force becomes $\frac{\int B^2 dA}{8\pi}$ dynes. The existence of this tension was mathematically demonstrated by Maxwell (*vide Electricity and Magnetism*, Vol. 2, Art. 642),

and the law was experimentally proved by Bosanquet (*vide Phil. Mag.*, Dec. 1886, and *London Electrician*, Vol. 18, p. 83). The evaluation of the tension can also be made by analogy from Thompson's *Electricity and Magnetism*, Art. 261, where the attraction between two charged plates is shown to be $\frac{V^2A}{D^2 8\pi}$. But $\frac{V^2}{D^2}$, by the analogy between electro-static and electro-magnetic phenomena, can be considered equivalent to $\frac{M^2}{(\rho l)^2}=B^2$. Hence the force between the plates is equivalent to $\frac{B^2A}{8\pi}$ dynes. The tension along lines of force is made manifest only where they pass from one material into another of considerably different permeability, and the theorem is therefore valuable in calculating the force with which an electro-magnet attracts its keeper. The formula is strictly exact only when the distance between the surfaces of magnet and keeper is small compared to their area, but it can be used with a fair degree of approximation in many useful problems.

From an inspection of the formula

$$F=\frac{\int B^2 dA}{8\pi}$$

it is evident that F may be increased by causing an uneven distribution of the induction over the area A, provided

$$N=\int B dA=BA$$

be kept of the same numerical value.

CHAPTER II.

SIMPLE ELECTRO-MAGNETS.

SURROUND a bar of iron by a coil of wire, through which an electric current flows, and the bar is magnitized. The arrangement of coil and bar (core) is called an **Electro-Magnet,** thus distinguishing it from the ordinary permanently magnetized steel bars, or **Permanent Magnets.** The magnetic properties of the electric current were first announced as recently as 1820, when Oersted published his investigations. Oersted's announcement led to a series of researches by Ampère, Arago, Faraday, Barlow, and their contemporaries (including Joseph Henry of Princeton College), which resulted in many valuable discoveries. Among the investigators was William Sturgeon, whom we have to thank for the invaluable discovery, made in 1825, that a bar of soft iron becomes magnetic when placed within a solenoid carrying an electric current, and that its magnetism is lost upon breaking the current. Many electromagnets were soon made and their effects were carefully studied by enthusiastic physicists, notwithstanding the difficulties to be overcome. At that day the laws of electric circuits were unknown, the common insulated wire of to-day was not made, and the manufacture of an electro-magnet was a matter of much labor. Moreover,

the only sources of current were, at first, plain zinc-copper cells, and later, Grove, Daniell, or similar types of galvanic cells. By the year 1845, the investigators had the great task set before them well in hand, and, overcoming their lack of experimental facilities, had mapped out the laws of magnetic circuits very much as we know them at the present time. This work may be said to have been completed by that marvellous scientist and engineer Joule, who gave his attention to electro-magnetic experiments between the years 1839 and 1850. During later years the mathematical and experimental work of Maxwell, Rowland, Bosanquet, Hopkinson, Kapp, and many others, has expanded and applied the vaguely understood results of the earlier investigators. (Consult Thompson's *Electro-Magnets; Enclyclopedia Britannica*, Arts. "Electricity," "Electro-Magnetism," and "Magnetism.")

It is now well to consider what occurs in a core when it is magnetized. The earliest theories which offered fairly complete explanations of the various phenomena of magnetism were those of Coulomb and Poisson; these were published before the discovery of the electro-magnet. They were followed by a host of theories which return more or less satisfactory results when put to the test of experiment. Since divided magnets always show two poles on each portion, however minute the division, the molecular nature of magnetism may be considered proven. It is therefore necessary for the theories to be founded upon some basis of molecular polarity, and the physical phenomena resulting therefrom. The theory of Coulomb (about

1785), which was extended and used by Poisson (about 1821), regarded all molecules as containing equal parts of two magnetic fluids (one "Austral," the other "Boreal"). When under the influence of a magnetic field, the fluids were supposed to separate and occupy opposite halves of the molecules. This hypothesis had many faults and was soon replaced by Ampère's theory (about 1830), (*vide* Maxwell's *Electricity and Magnetism*, Pt. 3, Chaps. 4, 6). In Ampère's theory, each molecule of magnetic material is supposed to be magnetized by an electric current which flows around it. When a bar of magnetic material is not polarized (*i.e.* is in the neutral state), the molecules are supposed to be arranged haphazard, but in such order as to neutralize each other's external effects. When the material is placed in a magnetic field, the molecules are swung around until their axes are parallel and their like poles are all pointing one way. In regard to this idea, Maxwell says: "If it should ever be experimentally proved that the temporary magnetization of any substance first increases, and then diminishes, as the magnetizing force is continually increased, the evidence of the existence of these molecular currents would, I think, be raised almost to the rank of a demonstration." (*Electricity and Magnetism*, Vol. 2, p. 436.) Ewing has shown that the intensity of magnetization of iron increases towards a definite maximum, but does not tend to decrease, within the limit of magnetizing powers practically attainable. Therefore the existence of electric currents circulating around the molecules seems to be doubtful. A theory that seems to more nearly approximate the true condition of the

molecules, was first advanced by Weber (about 1852), and was used by Maxwell in his mathematical investigations. In it, the molecules of magnetic matter are supposed to be polarized as an inherent attribute, exactly as gravitation is believed to be inherent in the molecules. That the theory can be made to cover quite satisfactorily most of the common experimental phenomena of magnetization has been shown by Hughes (*vide Proc. Roy. Soc.*, 1883, and *Jour. Soc. Tel. Eng.*, 1883, p. 374) and by Ewing (*vide Magnetic Induction in Iron and Other Metals*).

Hughes states the fundamental conclusions of the theory as follows:

1. "That each molecule of a piece of iron, steel, or other magnetic material is a separate and independent magnet, having its two poles and distribution of magnetic polarity exactly the same as its total evident magnetism when noticed upon a steel bar magnet.

2. "That each molecule, or its polarity, can be rotated in either direction upon its axis by torsion, stress, or by physical forces such as magnetism or electricity.

3. "That the inherent polarity or magnetism of each molecule is a constant quantity like gravity; that it can neither be augmented nor destroyed.

4. "That when we have external neutrality, or no apparent magnetism, the molecules, or their polarities, arrange themselves so as to satisfy their mutual attraction by the shortest path, and thus form a complete closed circuit of attraction.

5. "That when magnetism becomes evident, the

molecules, or their polarities, have all rotated symmetrically in a given direction, producing a north pole if rotated in that direction as regards the piece of steel, or a south pole if rotated in the opposite direction. Also, that in evident magnetism we have still a symmetrical arrangement, but one whose circles of attraction are not completed except through an external armature joining both poles.

6. "That we have permanent magnetism when rigidity, as in tempered steel, retains them in a given direction, and transient magnetism whenever the molecules rotate in comparative freedom, as in soft iron."

Granting the hypothesis of Weber, that the molecules are inherently magnets, it is evident that the smallest magnetizing force will swing them all completely around so that their axes are parallel if they are perfectly free to turn. Thus the slightest magnetizing force will develop the highest degree of magnetization. It is a fact, however, that magnetization goes on progressively with an increasing magnetizing force, which proves the existence of some force opposing the rotation of the molecules. Weber assumed such a controlling force to exist. Wiedemann and Hughes considered the molecules to be retarded in rotation by a sort of frictional resistance, and the latter regarded the resistance between certain narrow limits of rotation as smaller than for large angles of rotation. Weber's conception of a controlling resistance required considerable modification by Maxwell before it covered many of the phenomena of magnetization. The frictional theory served an excellent purpose, but it also could not account for

certain phenomena. Professor Ewing (about 1890) therefore replaced the frictional controlling resistance by the mutual reactions of the molecular magnets, and thus succeeded in explaining virtually all magnetic phenomena. He even succeeded in reproducing most of the important phenomena of magnetization by means of a mechanical model composed of many little magnetic needles (representing molecules) set at regular intervals upon a board. The variation in **Retentiveness** and **Coercive Force** of different specimens is explained by differences in the grouping of the molecules and in their distances apart. (Compare Ewing, *Magnetic Induction in Iron, etc.*, Chap. II.)

When a piece of magnetic material is magnetized, and the magnetizing force is then withdrawn, the magnetization of the material does not wholly disappear, but a certain degree of magnetization remains. This is called **Residual Magnetism**, and its magnitude is a measure of the **Retentiveness** of the test piece. Residual magnetism is held by different materials with various degrees of stability. In hard steel a large amount of knocking around may be required to materially reduce it, while the least jar will completely destroy it in soft iron. The force which holds the residual magnetism is called **Coercive Force** and is measured by the strength of the reverse field that is required to exactly remove all magnetization. The softest iron has the greatest retentiveness and the least coercive force, while hard steel with a smaller retentiveness has a very great coercive force. When the coercive force is great, the residual magnetism is called **Permanent Magnetism.**

The magnetism of the molecules has been verified by electro-depositing iron in a weak magnetic field. The iron thus deposited shows a considerably larger magnetic moment than would ordinarily be induced in it by a field of the same strength, making it probable that the molecules were swung into line, while separated from each other under the influence of the electric current.

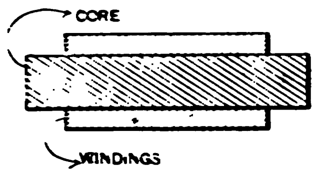


Fig. 8.

Silvanus P. Thompson classified the various types of electro-magnets according to their form as follows :

1. Bar magnet.
2. Horseshoe magnet.
3. Ironclad magnet.
4. Coil and plunger.
5. Special forms.

The first, second, and fourth types are very common. The first (Fig. 8) consists of a straight bar of iron sur-

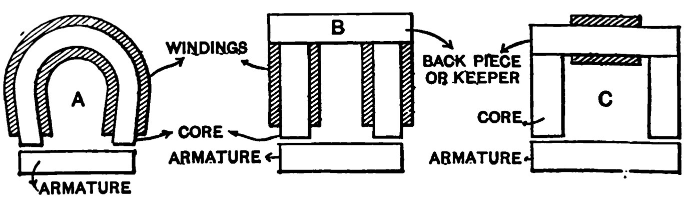


Fig. 9.

A, horseshoe with winding all over, core of one piece; *B*, horseshoe with winding on both legs, core of three pieces; *C*, horseshoe with winding on keeper, core of three pieces.

rounded by a coil or winding of insulated wire. In the second form (Fig. 9) the iron bar or core is in the form of a horseshoe. It may be in one piece or be made up of several, properly fastened together. The winding

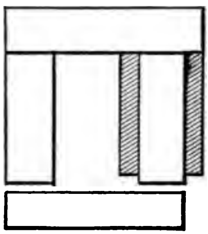


Fig. 10.

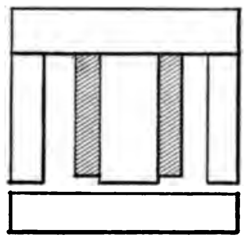


Fig. 11.

may cover the entire core, or only a part of it. When the winding is upon only one leg, as in Fig. 10, the magnet is called club-footed.

The third form (Fig. 11) is really a modification of the second, in which one leg is divided and is arranged to magnetically and mechanically protect the windings.

The fourth type (Fig. 12) is a simple coil without a core, into which a core may be sucked by magnetic action. This is sometimes made in the form of a modified ironclad magnet like *a*, Fig. 12.

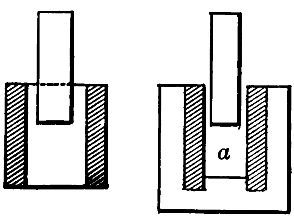


Fig. 12.

Simple electro-magnets are used for a large variety of purposes in various electrical manufactures. Examples: Coil and plunger, or horseshoe attracting armature in arc lamps; horseshoe attracting armature in telegraph

sounders and electric bells; bar magnet attracting disc in telephone receivers; horseshoe supporting load from attached armature; ironclad magnet in magnetic brake; and many others. The design for an electro-magnet to be used for most of these purposes is based upon the experience and "eye" of the designer. Thus, the conditions under which a telegraph sounder or relay is worked do not permit of exactly determining the total induction necessary to attract the keeper, and the size of the cores, or number of magnetizing turns, cannot be determined from the laws of magnetic circuits. Experience must be relied upon to determine what style of instrument is adapted to each class of work, but the laws of magnetic circuits can be used as a valuable aid in directing improvements (*vide* "Modern American Telegraph Apparatus," *Electrical Engineer*, 1892). In the case of a horseshoe magnet of considerable size which is to be used to give a fixed lifting or pulling power, the minimum dimensions can be at once determined from the magnetic laws, and the field magnets of dynamos are now invariably designed in accordance with the laws of magnetic circuits. In every case, the laws may serve an excellent purpose in directing the judgment of the designer.

Formerly the lifting power of a magnet was based upon its weight, though it was known that small magnets would support a greater proportional load than larger ones. As early as 1758 Bernouilli proposed a formula for the lifting power of magnets of similar form as follows:

$$F = a\,W^{\frac{2}{3}},$$

where F is the pounds of load supported, W the weight of the magnet in pounds, and a a numerical coefficient, the value of which depends upon the quality of the magnet. This is usually known as Haecker's rule, in honor of the man from whose experiments the rule was deduced.

The true formula for the lifting power of a magnet, which has already been developed, is

$$F = \frac{AB^2}{8\pi}.$$

This may be written

$$F = bA,$$

where b is a numerical coefficient, the value of which depends upon the quality of the magnet (*i.e.* the induction per square centimeter). In magnets of similar form, the polar surface A is evidently proportional to $W^{\frac{2}{3}}$, whence we have

$$F \propto bW^{\frac{2}{3}}.$$

This shows that Haecker's empirical rule, based purely upon experimental investigation, has a proper theoretical foundation, as was first pointed out by S. P. Thompson.

The coefficient a was given a value varying from 12.5 to 25 by the early experimenters. For modern horseshoe electro-magnets having a soft iron core the constant would be very much greater; but for present use Haecker's rule is not sufficiently exact on account of the change in the coefficient due to minor changes in the

form of the magnet. We, therefore, now go directly to the formula for tractive force,

$$F = \frac{AB^2}{8\pi} \text{ dynes} = \frac{AB^2}{8\pi \times 981} \text{ grammes,}$$

from which the total polar area in square centimeters that is required to enable an electro-magnet to support a load of F grammes upon its keeper, is

$$A = \frac{25000\,F}{B^2}.$$

In ordinary electro-magnets with a wrought-iron core of some magnitude, B will usually lie between the limits 10000 and 16000 lines per square centimeter.

The required polar area being determined, it should be increased by such a factor of safety as seems desirable for the purposes for which the magnet is designed.

It remains to determine the length of the core. According to the formula given on page 7, it is evident that an economy of magnetizing power is obtained by using a short core. Hence the core should be no longer than is demanded to accommodate the windings, unless outside mechanical conditions make this impossible.

Since the magnetizing power of a coil is proportional to the product $\overline{nc}$, the external conditions will usually prescribe the number of turns. Thus in an ordinary arc lamp the current is about ten amperes, and therefore the series regulating magnets must be made with sufficient turns to do their work with that amount of current. In the same way the windings on a telegraph instrument must be of such number that the required armature pull

will be given when the line current circulates. The currents used on telegraph lines are usually to be measured in milli-amperes (seldom exceeding 200 milli-amperes); hence the instrument must be wound with many turns of fine wire. A long telegraph line measures several hundred ohms, and a considerable resistance in the instrument does not entail a great proportional loss. On the other hand, where a coil carries a considerable current, the wire must be of considerable diameter and the windings of the fewest possible turns, to avoid an undue loss of energy in the coils and excessive heating. The area of the windings exposed to the air should not be less than two square inches for each watt lost in the coils if the maximum safe rise of temperature be assumed to be 75° F. This gives the relation

$$C^2R = CE = \tfrac{1}{2}S;$$

hence the maximum current that can be safely used in a coil of given resistance and surface is

$$C = .7\sqrt{\frac{S}{R}}.$$

Where windings have considerable depth, the inner turns become hotter than the outer ones. It is therefore necessary to avoid too deep a winding. S. P. Thompson states that in a coil wound to a depth of .5 of one inch, the wire can carry a current density of 3000 amperes per square inch, and where the depth does not exceed .3 of an inch the density can be increased to 4000 amperes per square inch, while the rise in temperature does not exceed 75° F.

If it be desired to wind two equal coils on bobbins, so that different currents will give the same rise of temperature in each, the following relations must be regarded.

Since the heating must be the same, $C^2R=C_1^2R_1$. But $R=\frac{\rho l}{D^2}$ and $R_1=\frac{\rho l_1}{D_1^2}$, where ρ is resistance per mil foot of copper, and l, l_1 are the lengths of wire on the bobbins given in feet, while D and D_1 are the diameters in mils (thousandths of an inch). The bobbins being equal, the volumes occupied by wire are equal; hence $l\propto\frac{1}{D^2}$ and $l_1\propto\frac{1}{D_1^2}$, from which $R\propto\frac{\rho}{D^4}$ and $R_1\propto\frac{\rho}{D_1^4}$. Therefore $\frac{R_1}{R}=\frac{D^4}{D_1^4}$, but $\frac{R_1}{R}=\frac{C^2}{C_1^2}$ which gives

$$\frac{C^2}{C_1^2}=\frac{D^4}{D_1^4} \text{ or } \frac{C}{C_1}=\frac{D^2}{D_1^2}.$$

If it be desired to wind a given spool to a certain depth and produce a given resistance, the diameter of the wire over the insulation is given in inches by

$$\delta=\tfrac{1}{2}i+\left[\tfrac{1}{4}i^2+\left(\frac{\rho l\,(D^2-d^2)}{R}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

where i is the thickness of the insulation (equal to the total diameter of the wire, minus the diameter of the copper), l the length of the spool, D and d the outer and inner diameter of the spool, all given in inches; ρ is the specific resistance of copper, and R the required resistance of the coil. The bare diameter of the wire is evidently $\delta-i$.

The length of wire, in feet, on any coil or spool, is ximately

$$L = .8 \frac{l}{\delta^2}(D^2 - d^2),$$

where δ is the diameter of the wire (insulated). The mean length of a turn on the coil is $\frac{\pi}{2}(D+d)$, the number of turns in a layer is $\frac{l}{\delta}$, l being the length of the spool, and the number of layers is $\frac{D-d}{2\delta}$. The total number of turns is then $\frac{l(D-d)}{2\delta^2}$ and the total length is

$$L = \frac{l(D-d)}{2\delta^2} \times \frac{\pi}{2}(D+d) = \frac{\pi l}{4\delta^2}(D^2 - d^2),$$

which is approximately as above $.8 \frac{l}{\delta^2}(D^2 - d^2)$. On account of the wires bedding into each other so as to increase the number of turns (which is not taken into account in deriving the formula), the approximation is probably as close as it is possible to come.

The energy lost in a coil being C^2R watts (proportional to the heat produced per second) and the magnetizing power being $\frac{4\pi nc}{10}$, the energy lost and the rise of temperature is the same for any winding, provided the size of the coil and the product nc be constant. Thus, if n be changed, the cross-section of the wire must be changed in inverse proportion, and the resistance will vary as n^2. From the stated conditions, C^2 varies inversely as n^2, hence $C_1^2R_1 = C^2R =$ a constant, and the heating effect remains unchanged.

For some purposes it is found convenient to use magnetized steel armatures for electro-magnets. These are called **Polarized Armatures.** Such an armature will

evidently be attracted by the magnet when the current passes in one direction, and be repelled when the current passes the other way. Polarized armatures are used in quadruplex telegraph instruments, telephone call-bells (where the ringing current is alternating), etc.

CHAPTER III.

THE MAGNETIC PROPERTIES OF IRON.

FROM the preceding pages it is evident that a full knowledge of the magnetic properties of iron is essential to the successful designing of electro-magnets. This applies with particular force to the design of the field magnets of dynamos, as they contain a considerable mass of iron, and it is desirable to compute with considerable exactness the windings required for a proposed machine. Unless the quality of the iron to be used is well known, the computations are likely to result in disappointment. In order to predetermine the dimensions and the windings of a magnet designed for a given duty, the most important magnetic constants to know are the relations of μ to B. It is usual to plot experimental determinations of their relations on cross-section paper, and the resulting curve, which is of the general form shown in Fig. 13, is called a **Permeability Curve**. For some purposes a curve, called the **Curve of Magnetization**, showing the relation of B to H is useful. Its char-

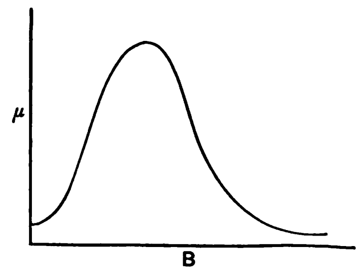


Fig. 13.

acteristic form is shown in Fig. 14. For small magnetizing forces the permeability is small, and the curve of magnetization begins with a short curvature convex to the x axis. As H increases, and μ grows rapidly larger, the curve of magnetization rises rapidly. When the permeability begins to grow less the curve of magnetization bends, as at A (Fig. 14), and the iron is said to be approaching **Saturation.** From this point, the curve continues indefinitely in a general direction not far from parallel to the x axis, and slightly concave towards it.

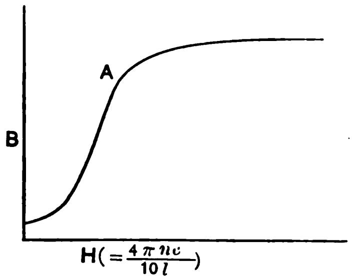


Fig. 14.

S. P. Thompson divides the methods of experimentally determining the points on these curves into four classes :

1. *Magnetometric class.*
2. *Balance class.*
3. *Inductive or Ballistic class.*
4. *Traction class.*

The first class covers experiments where the magnetization of a core is calculated from the deflection of a short magnetic needle placed at a fixed distance from it. This method was used by the early experimenters, such as Müller, Dub, etc. Professor Ewing also used the method in some of his experiments. On account of the difficulty of exactly determining H in the test piece, the method is not often used for practical tests. In order

to eliminate the effect of the ends of his test pieces, Professor Ewing was compelled to use wires of a length equal to 300 or more diameters. (See *Magnetic Induction in Iron and other Metals*.)

The second class is, in many respects, similar to the first class; but the deflection of the needle is balanced by known forces, or the deflection due to the difference between the magnetization of a known bar and of the test piece is taken. This method has been used very little. (See Hughes' "Magnetic Balance," *Jour. Soc. Tel. Eng.* Vol. 8.; Eickemeyer's "Magnetic Tester," *Transactions American Institute of Electrical Engineers*, Vol. 9; "Edison's Magnetic Bridge," *London Electrician*, Vol. 19.)

The third class is the one most used at the present time for determining the magnetic qualities of commercial iron. The class can be subdivided into the *Ring method*, *Bar method*, and *Yoke and Bar method*, according to the form of the test piece. Each of the methods depends upon measuring the transient electric pressure induced in a small test coil wound around the test piece, when the induction in the test piece is changed. This is usually done by a method invented by Weber, in which a **Ballistic Galvanometer** is used. This is a galvanometer with a rather heavy needle, and therefore a considerable time of vibration. If a current of short duration, compared with the time of vibration of the needle, be passed through the galvanometer, the coulombs of electricity which pass will be proportional to twice the sine of one half the angle of the first swing of the needle, as shown below. The first swing is often

called the "throw" of the needle. If the angular value of the throw be small the sine is sensibly proportional to the arc, and the quantity of electricity passed through the galvanometer is proportional directly to the throw. It is shown by Mechanics that an instantaneous couple which acts upon a torsion pendulum so as to cause rotation, is equal to the acquired angular velocity (ω) multiplied by the moment of inertia (I) of the pendulum. The needle of a ballistic galvanometer is a torsion pendulum which is acted upon by a couple equal to $GlmCdt$ (see page 17), when an instantaneous current passes through the galvanometer coil, and the needle is in the plane of the coil. Cdt is the number of coulombs passing through the galvanometer coil during the interval dt, and may be written Q. Equating these gives

$$GQlm = I\omega.$$

The energy of the needle when moving with the angular velocity ω is shown by Mechanics to be $\frac{1}{2}I\omega^2$, which is all expended in overcoming the couple due to the earth's field, while the needle swings through the angle θ to the turning point. Hence

$$\frac{I\omega^2}{2} = H_e ml(1 - \cos\theta) = 2H_e ml \sin^2 \tfrac{1}{2}\theta.$$

Substituting from the previous formula gives

$$Q = \frac{2}{G}\sqrt{\frac{H_e I}{ml}} \sin \tfrac{1}{2}\theta.$$

and this becomes, when θ is sufficiently small,

$$Q=\frac{\theta}{G}\sqrt{\frac{H_e I}{ml}}.$$

The time of a complete vibration of a torsion pendulum is shown by Mechanics to be $T=2\pi\sqrt{\frac{I}{D}}$ where D is the directive moment. This is $H_e ml$ in the case of a galvanometer needle, and therefore

$$T=2\pi\sqrt{\frac{I}{H_e ml}} \text{ and } \sqrt{\frac{H_e I}{ml}}=\frac{H_e T}{2\pi}$$

and finally

$$Q=\frac{H_e T}{2\pi G}\theta.$$

By Thompson's *Electricity and Magnetism*, Art. 394, $E=\frac{N_1-N_2}{10^8 t}n$, but $Q=Ct=\frac{Et}{R}$, where Q represents coulombs. Hence $Q=\frac{N_1-N_2}{10^8 R}n$. If the magnetization be reversed, N_1-N_2 becomes $2N$ and $Q=\frac{2Nn}{10^8 R}$. Whence $N=\frac{10^8 QR}{2n}$, and $B=\frac{10^8 QR}{2An}$. But $Q=K\theta$ where θ is the throw of the galvanometer, and K its constant (*i.e.* the coulombs per division of throw). This finally gives

$$B=\frac{10^8 RK\theta}{2An}.$$

Where R and n are respectively the resistance of the test circuit and number of turns in the test coil, A is the area of the cross-section of the test piece, and K and θ are respectively the constant and the throw of the galvanometer.

The ring method was first used by Rowland in 1872, in a classical series of experiments (see *American Journal of Science*, 1873, Vol. 6, and *Philosophical Magazine*, 1873). The general arrangement of his apparatus is shown in Fig. 15, where A is a ring made of the iron which it is desired to test, the ring being uniformly

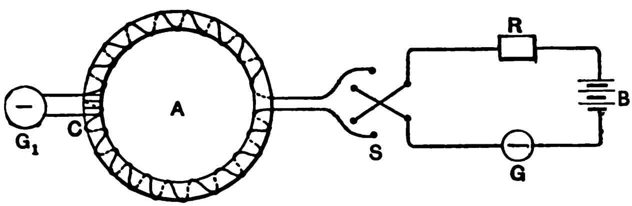


Fig. 15.

wound with a magnetizing coil; B is a battery or other source of current; G is an amperemeter; R is a variable resistance for controlling the magnetizing current; S is a reversing switch; and G_1 is a ballistic galvanometer, which is connected to the test coil C. The value of H is given by the formula developed on page 15,

$$H = \frac{4\pi nc}{10 l} = \text{approx. } 1.25\frac{nc}{l},$$

while B is found by the throw of the ballistic galvanometer when the magnetizing current is reversed. It is to be noted that $\frac{nc}{l}$ is the ampere turns per centimeter of length in the magnetic circuit, and in plotting the curve of magnetization, it is frequently more useful to make use of this term than of H (see page 49).

The ring method has been used by many experimenters, since Rowland first used it, including Bosanquet,

Ewing, Nichols, etc. It has the disadvantage, for commercial tests, of requiring a test piece in a form which is not always obtainable, and one of the following methods is therefore more useful.

In the bar method the test piece is a straight bar placed in a straight magnetizing solenoid. The value of B is calculated from the throw of a ballistic galvanometer connected to a test coil placed at the middle of the bar. The difficulty encountered in the first class, in the exact determination of H at the middle of the bar, is found here. The method has been used by a number of experimenters in special studies, or for checking the ring method. It is quite convenient for use, as the solenoid is permanent, and various cores are easily slipped into place.

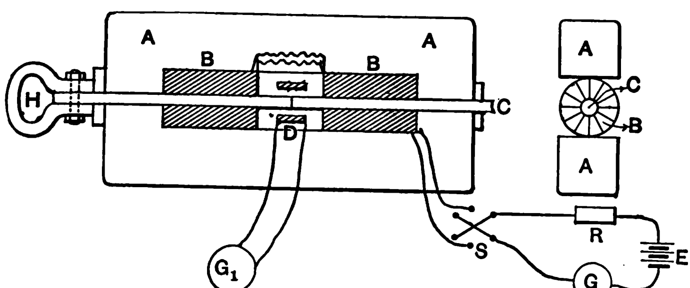


Fig. 16.

To avoid the difficulty in determining H, and retain the convenience of the bar method, Dr. John Hopkinson devised the yoke and bar method (*Philosophical Transactions*, 1885, p. 455, and Thompson's *Lectures on the Electro-Magnet*). The general arrangement of the apparatus is shown in Fig. 16, where A is a well

annealed, heavy forging, with a rectangular space for the magnetizing coils *B*, and *C* is the test piece, which is in two parts, with the abutting faces carefully surfaced. The test piece passes through the ends of the forging with a close fit. One end is fastened rigidly, while the other has a handle *H*, by means of which it can be withdrawn a short distance. *D* is the test coil, which is arranged with a spring so as to jump out of the field when one part of the test piece is slightly withdrawn by pulling the handle; *G* is an amperemeter; *S* is a reversing switch; *E* is a source of current; *R* is a variable resistance, and G_1 is a ballistic galvanometer.

In the apparatus, as used by Hopkinson, the forging, or yoke, was about 18″ long, $6\frac{1}{2}$″ wide and 2″ thick. The test pieces were $\frac{1}{2}$″ in diameter, and of the proper length to reach through the yoke. The magnetizing coils contained a total of 2008 turns, and the test coil contained 350 turns. As the magnetizing current is not reversed in Hopkinson's method, but the test coil is quickly removed from the field, the induced electric pressure is due to *N*, and the numeral 2 in the denominator of the formula for *B* disappears. It is evident that the value of *B* can be deduced by observing the swing of the galvanometer, when the magnetizing current is reversed, as in the ring method, in which case the test piece need not be divided.

The test coil used in the Hopkinson method has a considerably larger mean area than the cross-section of the test piece, and the number of lines of force passing through the coil is therefore greater than the number passing through the test piece. The correction which

must be applied to the indications of the ballistic galvanometer, to determine N in the test piece, can be obtained by directly measuring the mean area of the coil, or, better, by indirectly determining its area from the swing of the galvanometer, with a copper or wooden rod in the place of the test piece.

The yoke and bar method eliminates the end effect, which is objectionable in the bar method, by completing the magnetic circuit through iron; but the magnetic circuit is not uniform, and therefore the formula for H is not quite as simple as in the ring method.

Let L be the mean length of the lines of force in the test piece;
L_1, their mean length in the yoke;
B and B_1, the values of the induction in the test piece and yoke respectively;
μ and μ_1, the respective permeabilities;
and A and A_1, the sectional areas of test piece and yoke.

The magnetic circuit formed by the test piece and yoke is so complete that very few lines of force will leak through the air. The total induction in different parts of the circuit can therefore be considered as constant, thus:

$$N = BA = B_1 A_1;$$

and since $B = \mu H$, and $B_1 = \mu_1 H$,

there results $H = \frac{B}{\mu} = \frac{B_1}{\mu_1}$.

Also, $M = \frac{4\pi nc}{10} = \int_0^l H dl = HL + HL_1$.

which gives $M = \frac{BL}{\mu} + \frac{B_1L_1}{\mu_1} = N\left[\frac{L}{\mu A} + \frac{L_1}{\mu_1 A_1}\right]$
$$= N(P + P_1),$$

in which P and P_1 are the reluctances of test piece and yoke. If the yoke be sufficiently large compared with the test piece, its reluctance can be considered negligible, and the formula becomes

$$M = \frac{4\pi nc}{10} = \frac{NL}{\mu A} = \frac{BL}{\mu} = HL,$$

and $$H = \frac{4\pi nc}{10L}.$$

This has the same form as the value of H given for the ring method. The effective length L of the test piece, cannot be measured exactly because the test pieces extend through the yoke, and the exact distribution of the lines of force at the ends is not known. If it be taken as the distance between the ends of the yoke, centre to centre, the approximation is sufficiently close.

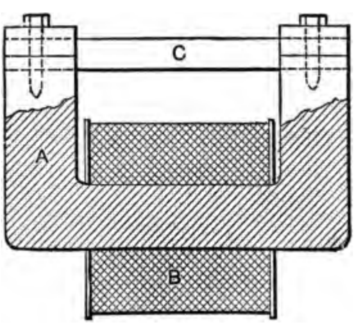


Fig. 17.

Mr. Burton has made a modification of the Hopkinson apparatus for some tests in the University of Wisconsin laboratory. The arrangement is similar to one half of the Hopkinson apparatus, with the magnetizing coil on the yoke instead of on the test piece. As shown in Fig. 17, A is a heavy forged

yoke of the best Swedish iron, *B* is a magnetizing coil of 1000 turns, and *C* is an accurately turned test piece. The ends of the test piece are clamped in accurately bored holes as shown, by caps and screws, as shown in Fig. 18. Here the formula for *H* is the same as in the Hopkinson arrangement, provided the reluctance of the yoke is sufficiently small in comparison with that of the test piece. The form of yoke and position of the magnetizing coil cause a considerable leakage between the two legs of the forging, which increases the total induction in the yoke, and makes it necessary in some cases to correct the calculated value of *H*. The value of the induction in the test piece is found by a test coil and ballistic galvanometer. Instead of using the test coil and ballistic galvanometer, a spiral of Bismuth wire, as made by Hartmann and Braun, may be placed in the magnetic circuit of the yoke, in which case the value of *N* can be directly determined from the electrical resistance of the spiral.

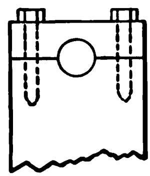


Fig. 18.

The fourth class covers methods of measuring *B* by observing the tractive force at a joint in the iron core. Thus, if a ring of iron be divided into two semi-rings, and each half be uniformly wound with a magnetizing coil, the value of *B* can be calculated from the force required to pull the halves apart. *H* can be calculated from the current and the windings, in the usual manner. Instead of a ring, a rod divided at the middle may be used.

S. P. Thompson has devised a form of apparatus which he calls a pereameter, for using this method

in commercial tests. The arrangement is shown in Fig. 19, where A is a heavy forging, B is a magnetizing coil within which is a brass tube, and C is a test piece which is slid down the tube. The joint at d, between test piece and yoke, is carefully surfaced. From the force required to pull the test piece loose from the yoke at d, the value of the induction is readily determined. The value of H for any magnetizing current is determined by the same formula as in the Hopkinson apparatus.

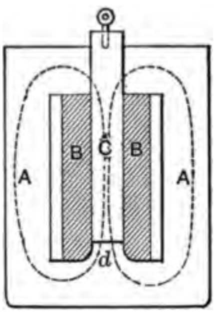


Fig. 19.

To determine B from the swing of the ballistic galvanometer, the constant of the galvanometer (*i.e.* the instantaneous flow in coulombs required to give a throw of one division) must be known.

This can be determined by the following methods :

a. *By discharge of a standard condenser.* If a condenser of P microfarads capacity be charged by E volts pressure, the charge is $Q=\frac{PE}{1000000}$ coulombs. If the charge be passed through a ballistic galvanometer, giving a scale reading of θ divisions, the constant is evidently $K=\frac{PE}{1000000\,\theta}$

b. *By vibration and deflection.* The formula $Q=\frac{H_e T}{2\,\pi G}\theta$ contains only known or determinable constants in the right hand member. T may be readily determined by timing the vibrations, and $\frac{H_e}{G}$ may be determined by

passing a measured continuous current through the galvanometer and noting the steady deflection θ_1. From the formula for the tangent galvanometer given on page 16, it is evident that $\frac{H_e}{G}=\frac{C}{\theta_1}$, when θ_1 is sufficiently small.

Substituting this value of $\frac{H_e}{G}$ in the formula

$$Q=\frac{H_e T}{2\pi G}\theta$$

gives
$$K=\frac{Q}{\theta}=\frac{TC}{2\pi\theta_1}.$$

c. *By standard earth coil* (Rowland's method). A coil of area A when lying on a horizontal table encloses $V_e A$ lines of force when V_e is the vertical component of the earth's magnetism. If the coil be made up of n turns of wire, and it be quickly turned over, the average electric pressure produced is $\frac{2nAV_e}{10^8 t}$. The number of coulombs flowing through any circuit connected to the coil is $\frac{Et}{R}$, where t is the time occupied in reversing the coil and R is the resistance of the circuit. This becomes by substitution $\frac{2nV_e A}{10^8 R}$, and hence $K=\frac{2nV_e A}{10^8 R\theta}$.

d. *By standard solenoid* (probably suggested by Lord Kelvin). In a long solenoid of known constants, the total induction caused by a current C is $N=\frac{4\pi AnC}{10l}$. If a small secondary (test) coil of n_1 turns be wound upon the solenoid, $E=\frac{2Nn_1}{10^8 t}$ volts are induced in it upon reversing the current in the solenoid.

If the test coil be connected to a ballistic galvanometer, the coulombs are as before,

$$Q = \frac{Et}{R} = \frac{2\,Nn_1}{10^8 R},$$

and

$$K = \frac{2\,Nn_1}{10^8 R\theta} = \frac{8\pi A C n n_1}{10^9 l R\theta}.$$

A direct method of measuring B has just been suggested by Siemens and Halske, who manufacture an

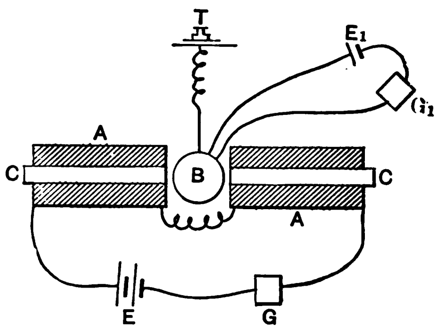


Fig. 20.

instrument in which the method is applied, (Fig. 20). AA are two magnetizing coils, into which test pieces CC can be slipped. B is a swinging coil connected to the torsion head T. E and E_1 are sources of current connected to the magnetizing and swinging coils respectively, and G and G_1 are amperemeters in the respective circuits. When fixed currents are passed through A and B the latter experiences a certain turning force when no test pieces are in AA and a greater turning

force when the test pieces are in place. The ratio of the two forces is roughly proportional to $\frac{F_1}{F}=\frac{B}{H}=\mu$, F_1 being the force when the test pieces are used. On account of the shortness of the test pieces, this instrument is not likely to be a satisfactory one, but it possibly might be modified so as to obtain satisfactory results.

The formula giving the value of H in terms of the dimensions of a coil makes it evident that the ampere turns nc, or the ampere turns per centimeter $\frac{nc}{l}$, may be used instead of H in plotting curves of magnetization. By using proper scales, it is evident that the curves will be exactly alike when plotted with H, nc, or $\frac{nc}{l}$. The latter is a very convenient constant for practical use. If the ampere turn be taken as the unit of magnetic pressure, we have $M = 1.25\,\overline{nc}$ and $H = \frac{1.25\,\overline{nc}}{L}$, where 1.25 is $\frac{4\pi}{10}$ approximately, and L the length of the magnetic circuit.

It has already been shown that it is necessary to know the permeability curve, or curve of magnetization, of the iron from which an electro-magnet is to be made, before a design can be successfully predetermined. For commercial economy of manufacture, it is also necessary to know the relation between the cost and the quality of the various samples which may be available for use. The magnetic quality of iron depends upon the following elements :

1. Its physical condition (temper, homogeneity, etc.).
2. Its chemical composition.
3. Some uncertain molecular, or physical conditions, about which little is known.

The comparative magnetic qualities of various samples can be determined at once by a comparison of their curves. By a consideration of the magnetic qualities, the comparative cost of working in the shop, and the first cost of metal, the relative commercial economy of different samples is determined.

Physical conditions due to the method of manufacture, and to the after treatment of iron, have a marked effect upon the permeability curve; and it is not unusual to find somewhat different magnetic qualities in two test pieces cut from the same forging, or cast from the same foundry charge. Thus, among several samples of one brand of wrought-iron tested by Bosanquet, the maximum permeability varied from about 1900 to 2500. Again, dynamo frames made of cast-iron often show a difference of permeability, although they are apparently produced by uniform treatment of uniform material. In the case of cast-iron frames there may be a difference in the chemical composition due to different temperatures at pouring, which partially accounts for their different qualities. Differences in the magnetic quality of two similar samples are likely to be greater near the maximum point of μ than at the values of B used in common practice. The accompanying curves (Fig. 21) show the effect that can be produced on the permeability curve by controlling the physical properties. Curves A and A' were taken from a sample of mild steel cast-

Permeability Curves showing effect of changing Physical Properties.

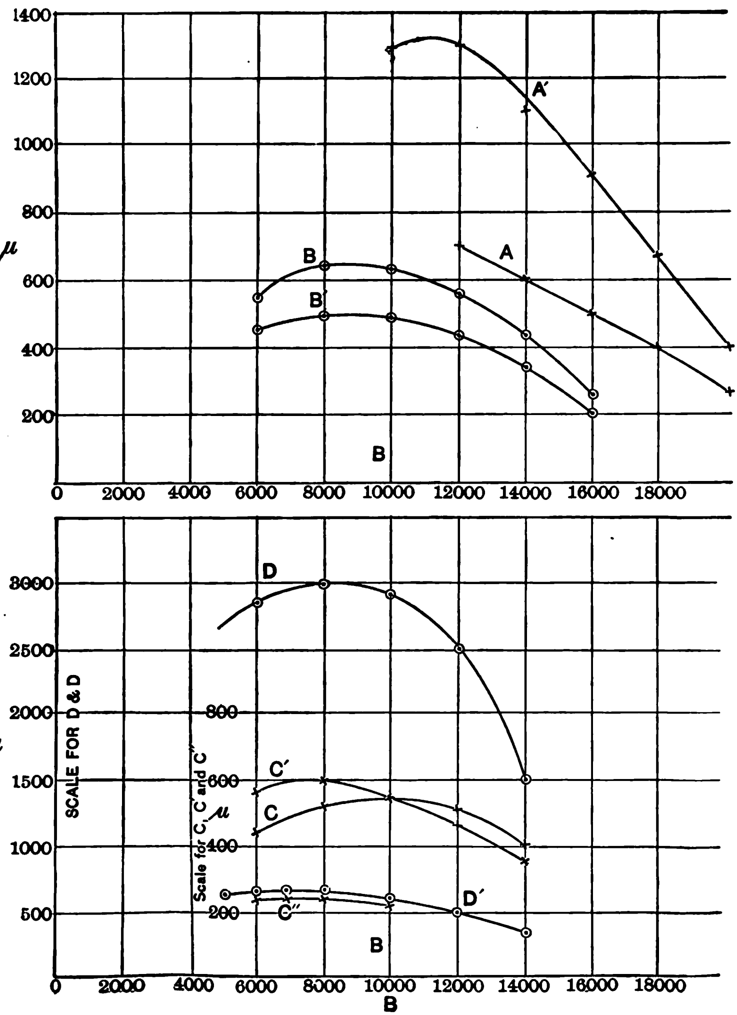


Fig. 21.

ing, forgeable but not capable of taking a temper. Curve *A* shows part of the permeability curve for the sample as cast, and *A'* shows it after the test piece had been thoroughly annealed. Curves *B* and *B'* were taken from a sample of mild steel capable of taking a temper. *B* shows part of the permeability curve for the test piece after thorough annealing, and *B'* after tempering in oil. *D* and *D'* are the curves for a soft iron wire, first when carefully annealed, and second when hardened by being subjected to sufficient strain to stretch it ten per cent in length. Chilling cast-iron in the mould has an effect similar to that caused by the hardening in the latter sample. The chemical change due to chilling, however, probably plays the most important part in this case.

The importance of avoiding all chilling or hardening (by cold rolling, hammering, etc.) during the processes of manufacture, is clearly shown in these examples; not only is the permeability curve lower for the hardened metal, but its maximum is almost always at a smaller induction. For use in dynamo magnets, it is desirable that the permeability be as great as possible at large inductions, and the maximum should therefore come as late in the curve as possible.

The sample from which the curves *A*, *A'* were taken, is an admirable example of the mild steel castings that are now used to a considerable extent for dynamo frames. Ordinarily, the use of these castings, in the place of cast or wrought iron, is likely to effect an economy in dynamo construction on account of their excellent magnetic qualities at high inductions, the ease of work-

ing them in the shop, and their comparatively low cost. They have the disadvantage, to be found in all steel castings at present, that sound metal in large masses is difficult to obtain. How far the process of annealing steel castings can usually be carried with economy, must depend upon local conditions; but when not annealed the castings possess a permeability for values of B between 14,000 and 20,000, which is fully equal to that of good merchant wrought-iron (see Henrard, *La Lumiere Electrique*, Vol. 33; and Thompson, etc., *Transactions American Institute of Electrical Engineers*, Vol. 9, p. 250).

The effect of high temperatures upon the permeability of iron and steel is remarkable. For small magnetizations, the permeability increases to a maximum, and then suddenly drops to nearly unity when a certain temperature is reached. This temperature is called the **Critical Temperature,** and seems to vary between the limits of 650° C. and 900° C., the exact value depending upon the nature of the test piece. For magnetizations as large as those used in ordinary practice, the changes in permeability due to changes in temperature are not so rapid; the permeability does not vary to any marked extent, as the temperature rises beyond the ordinary temperature of the air, until the critical point is approached, when the permeability begins to fall gradually, and finally reaches a value near unity. The curves x, y, and z, Fig. 22, plainly show the change of the permeability with temperature changes. They were taken from a test piece of soft iron; first, when subjected to the very small magnetizing force of three-tenths of a C.G.S. unit (curve x), second, when subjected to a

force of 4 C.G.S. units, and third, when subjected to a magnetizing force of 45 units, which is a value within the range of practical use (curve *z*). (*Vide* Hopkinson, *Philosophical Transactions*, 1889; Ewing, *Magnetic Induction in Iron, etc.*) The peculiar effect of temperature upon the permeability of steel, containing a considerable percentage of nickel, is reverted to later (page 56).

The chemical composition of mild steels has an enormous influence upon their magnetic qualities. Their sensitiveness to differences of treatment may sometimes mask the effects of variations in the composition; but the following general statements can be tentatively made as covering our present knowledge of the effect of chemical impurities, within the range of inductions used in dynamo magnets:

1. The permeability depends inversely upon the amount of carbon present.

2. The permeability depends inversely upon the amount of manganese present when it much exceeds .15 per cent. Small percentages of manganese seem to have a greater effect for large values of B than for small values. The presence of 3 or 4 per cent of manganese decreases the value of μ for all inductions in a very marked manner. With 12.5 per cent of manganese present, the permeability of steel is practically constant at 1.4. Hopkinson says of 12.5 per cent manganese steel, "the induction is strictly proportional to the magnetizing current (*i.e.* μ = constant), and the material shows no loss by hysteresis. . . . Smaller proportions of manganese reduce the magnetic property in a less degree, the reduction being greater as the quantity of manganese is greater."

3. The effect of less than .5 per cent of silicon does not seem to be marked, but when present in larger percentages it evidently decreases the permeability.

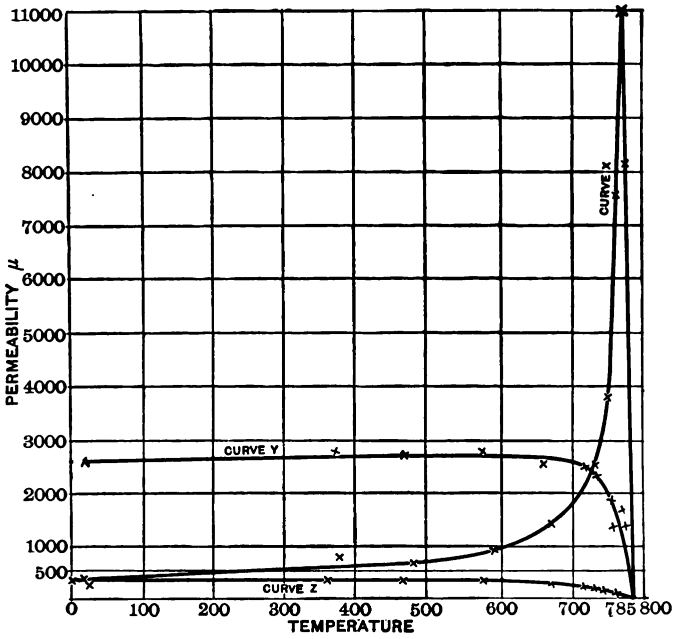


Fig. 22.

When in small quantities and in the presence of manganese, it possibly tends to counteract the hurtful effects of the latter.

4. The amount of phosphorus and sulphur present in forgeable steel castings is too small to have any marked effect on the permeability.

5. Nickel, chromium, tungsten, etc., seem to have an injurious effect even when present in small quantities.

Tungsten is used to increase the coercive force of permanent magnets, but the permeability of such magnets is always low. The coercive force of soft wrought iron is usually not much greater than 2; in chrome steel, oil tempered, it may reach 40; and in tungsten steel it may be greater than 50. Nickel in large percentages has a most peculiar effect on steel. Of 25 per cent nickel steel, Hopkinson says: "It is non-magnetic as it comes from the manufacturers. Cool it, however, a little below 0° C., and it becomes very decidedly magnetic. But if now the cooled material be allowed to return to the ordinary temperature, it is magnetic. If it be heated, it is still magnetic till a temperature of 580° C. is attained, about which it becomes non-magnetic. Now cool it, and it remains non-magnetic till a temperature of a little less than zero is reached, when it becomes magnetic again."

6. Steels cast by an aluminum or silicon process frequently, if not usually, have a high permeability. This may be due to the aluminum or silicon alloying with the iron; but as these elements do not seem to remain in the product to any considerable extent, the high permeability is probably due to the greater homogeneity caused by their fluxing qualities.

The effect of carbon on the permeability of steel is plainly shown in the accompanying curves *E* and *F* (Fig. 23). *E* is the curve of a steel casting capable of welding and tempering, and carrying C=.40, Si=.08, Mn=.18, P=.04, S=.017. *F* is the curve of a steel casting capable of welding and tempering, carrying C=1.13, Si=.09, Mn=.19, P=.04, S=.015.

Permeability Curves showing the Effect of Various Impurities in Mild Steels and Wrought Iron.

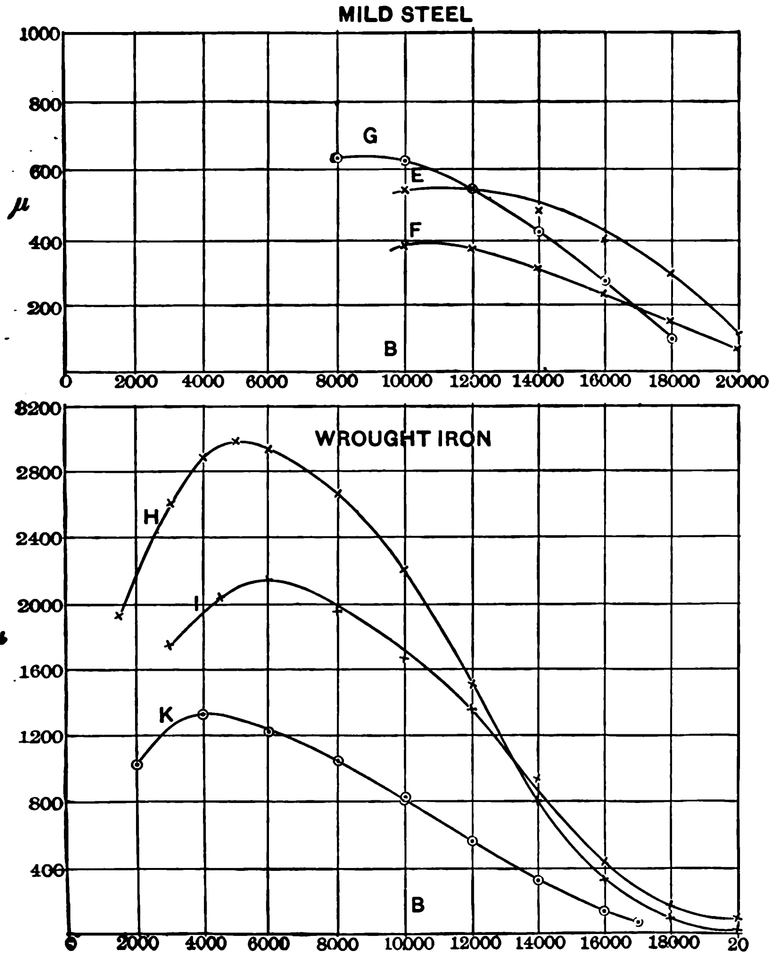


Fig. 23.

The effect of manganese is shown by curves *G* and *E*. It is to be noted that the test piece of curve *G* carried considerably less carbon than that of curve *E*, but it shows a lower permeability curve, probably on account of the predominating influence of its greater percentage of manganese. Curve *G* represents a test piece of Whitworth's mild steel, with C=.32, Si=.04, Mn=.44, P=.035, S=.017.

The effect of silicon in large percentages is shown by a sample tested by Hopkinson. It carried C=.685, Si=3.44, Mn=.69, P=.13, S=.02, and was carefully annealed before testing. Its permeability, with B a little under 15,000, was about 60. The comparatively large amount of phosphorus in this piece may have aided somewhat in reducing the permeability.

Impurities in wrought iron seem to have an effect on its permeability similar to their effect on the permeability of steels. Owing to its greater purity and its excellent physical properties, the permeability of wrought iron has a much higher maximum than is shown by steels. Rowland found the maximum value of μ to be 4600 at $B=5400$ in an excellent piece of Norway iron, while μ was 350 when B was 17,000. Characteristic permeability curves for Swedish iron and ordinary wrought iron are shown in *H* and *I* (Fig. 23). Curve *H* is the average taken from three test pieces carrying approximately C=.08, Si=.03, Mn=.01, P=.03, S=.01. Curve *I* is the average taken from three test pieces of a good quality of merchant wrought iron, carrying approximately C=.075, Si=.10, Mn=.25, P=.10, S=.10.

The effects of manganese, and possibly of phos-

phorus and sulphur, are apparent in the smaller maximum permeability exhibited in curve *I*. The greater permeability shown by curve *I* for the large values of *B* makes it seem possible that the effects of Si, P, and S in small quantities is to increase the permeability for large inductions. This may also be true for steels. The cause of this difference in the curves is likely, however, to be due to physical conditions, as soft wrought iron is specially sensitive to differences of treatment. Ewing found that the maximum permeability of a very soft iron rod could be increased by tapping to the enormous value of 20,000, while under ordinary conditions it showed a curve similar to that usually given by good wrought iron. Hence, differences in the methods of testing, or some vibrations, may possibly explain the anomalous differences in the forms of curves *H* and *I*.

As far as our knowledge extends, we may say that it is probable that impurities affect the permeability of wrought iron in the same manner as they affect mild steels. The permeability attains a much greater maximum in wrought iron, and the maximum usually occurs at a materially smaller value of *B*. For inductions greater than 15,000, the permeability seems to be fully as great in good forgeable steel castings as in wrought iron. Ewing says: "Speaking generally, the curves of magnetization for steel can be made to closely resemble those for iron by simply altering the scale of *H*. Under strong magnetic forces, the region of saturation is reached in steel with much the same value of *I*, or of *B*, as in iron; but to reach it or to remove it, requires the application of a stronger force." That the coercive

force of mild steel is nearly always greater than that of wrought iron is shown by the complete curves of magnetization (see page 73).

As a finished dynamo frame made of a mild steel casting should ordinarily cost less than one made largely of forged wrought iron, the economy of steel seems unquestionable, provided uniform homogeneous castings can be obtained.

Aluminum in wrought iron, as used in the "mitis" casting process, seems to have the simple function of a flux, which is required to make a homogeneous casting. Very little aluminum remains in the product, and the casting has physical and magnetic properties similar to those of ordinary forged wrought iron. Curve *K* (Fig. 23) shows the permeability of a sample of "mitis" casting. That the small amount of aluminum remaining in the iron acts to injure its magnetic quality is shown to be probable by a comparison of curve *K* with curves *H* and *I*, though here again the difference in treatment may be the controlling cause of the differences in the curves.

The treatment that cast iron receives in the cupula and in casting may vary its impurities over a wide range of percentages. Very decided variations may exist in the iron poured from a single charge. It is thus doubly important to study the effect of foreign elements upon the permeability of cast iron, in order that both the material used in the foundry, and the treatment it receives, may be of such a nature as to give the best magnetic characteristics to the iron. The frames of continuous current dynamos are not usually

made of cast iron alone, but cast iron is frequently used for pole pieces, bed plates (serving as keeper), etc.; while the cores (under the windings), or cores and keeper, may be made of wrought iron. Nevertheless, some very excellent continuous-current machines have frames wholly of cast iron, and it is usual to make the frames of alternating-current machines wholly, or largely, of cast iron. The working permeability of cast iron is low under the best conditions, and a poor quality is to be avoided if possible. A poor grade of cast iron used for dynamo magnets must result in machines that have an excessive weight and cost.

The effects of impurities upon the permeability of cast iron can only be tentatively stated, although they are better known in this case than in the case of steel and wrought iron.

1. The permeability depends inversely upon the total amount of carbon present.

2. With a fixed percentage of total carbon, the permeability depends inversely upon the ratio of combined to graphitic carbon. Thus, gray and white iron containing the same total constituents will give very different permeability curves, that for the gray iron being higher. In malleablizing cast iron, the total carbon is decreased towards a limit of less than .1 per cent combined carbon, while the other impurities are practically unchanged. The process also serves to thoroughly anneal the material. The marked decrease in carbon and the annealing causes thoroughly malleablized iron to give a permeability curve which is decidedly superior to that given before malleablizing. Chilling in the

mould changes graphitic carbon into combined carbon, besides causing changes in the physical structure; hence it lowers the permeability. Neither malleablizing nor chilling can be fully carried out on the large masses of ordinary dynamo frames, but the softness and homogeneity of castings produced from a foundry heat depend largely upon care in founding.

3. The presence of manganese has a harmful effect upon the magnetic quality of cast iron. This is probably due mainly to its tendency to cause chilling (*i.e.* change of graphitic into combined carbon), but it may have an additional effect similar to its influence on wrought iron and steel.

4. Silicon in percentages not exceeding 2.5 per cent seems to be advantageous, as it tends to make the iron soft and prevent chilling. Howe says of silicon: "It diminishes the power of iron to combine with carbon, not only when molten (thus diminishing the total carbon content), but more especially at a white heat, thus favoring the formation of graphite during slow cooling; it lessens the formation of blow-holes; it hinders at high temperatures the oxidation of iron and probably of the elements combined with it."

5. Sulphur and phosphorus have a tendency to cause chilling, and thus decrease the permeability of cast iron.

6. Aluminum seems to increase the homogeneity of cast iron, and to affect the contained carbon in much the same manner as does silicon, while in some grades of iron the effect is much more pronounced. The effect of aluminum upon cast iron is greater than upon steel, on account of its direct action in controlling the carbon,

but its usefulness depends upon the quality of the pig iron used. Thus the addition of aluminum to a soft gray foundry iron results in loss of strength in the product, because the fluxing qualities of the aluminum are not needed, and it therefore acts merely as an impurity. On the other hand, its addition to certain grades of iron may make a marked improvement in the product.

The effect of varying the carbon is plainly shown in the accompanying permeability curves *L* to *S* (Fig. 24). Experimental data are wanting to fully show the effect of silicon, manganese, etc.

Curve *L* is part of the permeability curve of a hard white cast iron, carrying approximately the impurities: Graphitic carbon = 0, combined carbon = 2.5, Si = .8, Mn = .8, P = .5, S = .3.

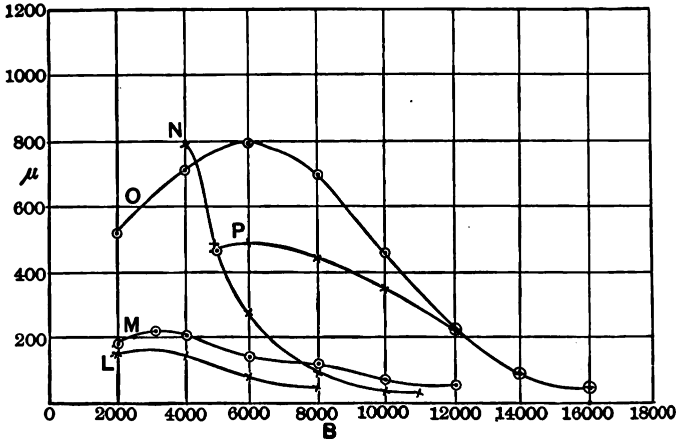
Curve *M* shows the average of several test pieces of medium gray iron carrying approximately: Graphitic carbon = 2.5, combined carbon = 1.0, Si = 2.0, Mn = .3, P = .1, S = .05.

Curve *N* is the result of a test by Hopkinson of a soft gray iron. It must be very low in carbon and manganese.

Curve *O* is the average given by a number of test pieces made of malleable cast iron which carried about the same impurities as the test piece of curve *M*, with the exception of the carbon. The latter probably did not exceed .1 per cent or .2 per cent, all in the combined form.

Curve *P* was taken from a test piece of commercial malleable iron of unknown composition.

Permeability Curves showing the Effect of Various Impurities in Cast Iron.



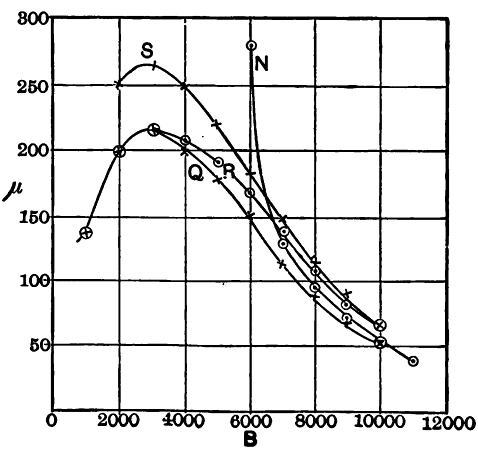


Fig. 24.

Miscellaneous Curves.

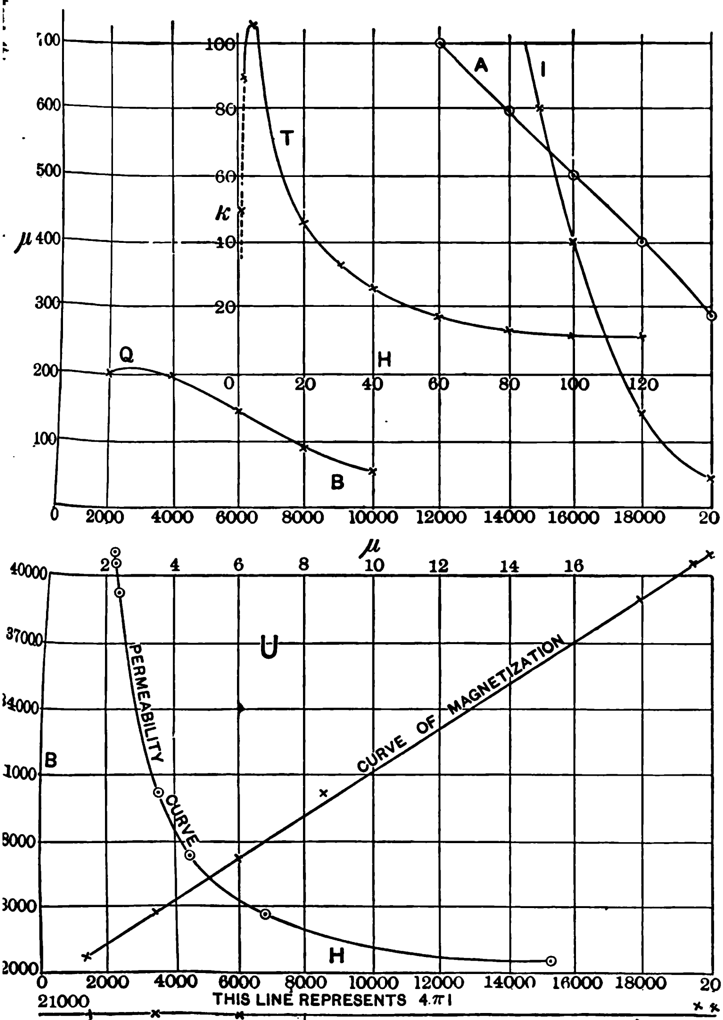


Fig. 25.

A good gray cast iron used by one of the large manufacturers of dynamos gives the curve *Q*. This carries about .06 per cent of aluminum. The same iron, with an admixture of aluminum of 1.4 per cent and 6.5 per cent, gives curves *R* and *S*. When the aluminum is increased to 12 per cent, the permeability curve falls below curve *S*. (See *Magnetic Induction in Iron and Other Metals*, Hopkinson, *Philosophical Transactions*, 1885, and *Journal Institution of Electrical Engineers*, 1890; Henrard, *La Lumière Electrique*, 1886; Rowland, *American Journal of Science*, 1873; Howe, *Metallurgy of Steel*.)

Curves *A*, *I*, and *Q* are shown together upon the accompanying sheet (Fig. 25). These show very well the characteristic forms of the permeability curves of good mild steel, merchant wrought iron, and good gray cast iron.

A curve showing the relation between I and H must evidently be similar in form to the curve of magnetization, but will fall below it, since ordinates of the I-H curve are $\frac{B-H}{4\pi}$. In the same way a curve showing the relation of κ and B is similar to the permeability curve, but much flatter, for the ordinates are $\frac{\mu-1}{4\pi}$. The curve T (Fig. 25) is a typical one showing the relation of κ to H. The rapid increase of κ for small values of H is to be remarked, and also the peculiar hyperbolic form of the second branch of the curve with one leg apparently asymptotic to the axis of X. With this form of the susceptibility curve, it is evident that the curve representing the relation between I and H should

become parallel or nearly parallel to the axis of X. This has been virtually proven by Ewing's experiments, in which he used magnetizing powers from $H=1500$ to $H=20,000$. In summing up the results of his experiments, Ewing says: "Under sufficiently strong magnetizing forces the intensity of magnetization, I, reaches a constant, or very nearly constant, value in wrought iron, cast iron, most steels, nickel, and cobalt. The magnetic force at which I may be said to become practically constant is less than $(H=)$ 2000 C.G.S. units for wrought iron and nickel, and less than $(H=)$ 4000 for cast iron and cobalt. In stronger fields the relation of magnetic induction to magnetic force becomes

$$B=H+4\pi I=H+\text{constant."}$$

For wrought iron the constant is nearly $4\pi\times 1700$, and for cast iron it is about $4\pi\times 1240$.

The accompanying curve U (Fig. 25) shows the permeability curve and curve of magnetization for large values of H, in a sample of Swedish Lancashire iron.

Figure 26 shows the relation of $1/\mu=\rho$ to H, and is interesting from a scientific point of view.

If a neutral piece of iron or other magnetic material be magnetized and the magnetic pressure then removed, a certain proportion of the total induction (already defined as residual magnetism) remains. If a larger magnetic pressure be applied, the magnetization reached will be greater than before, and if the magnetic pressure be reduced to its former value, the total induction will not decrease proportionally, but will continue at a higher value than was previously attained with the same

magnetic pressure. It may be generally stated that the total induction attained by any test piece, when subjected to a magnetic pressure, depends upon its previous magnetic state. The ordinates of a curve of magnetization therefore depend, to some degree, upon the method used in testing. If the curve be first taken

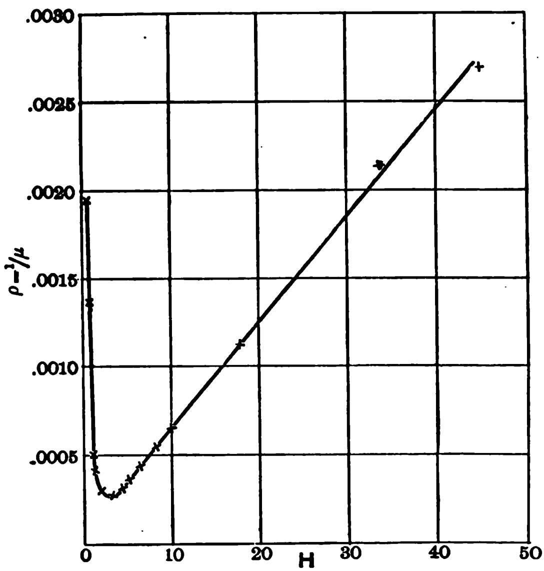


Fig. 26.

by measuring the increments of induction due to a step by step increase of H, and then those due to a step by step decrease of H from its maximum value, two curves will be the result, as shown in Fig. 27, where OR is the up (or increasing) curve, and RS the down (or decreasing) curve. This phenomenon, a knowledge of which is of much importance to the designer, was named by Ewing,

Hysteresis. Ewing also proved that if a certain magnetic pressure be applied to a test piece, the magnetization at once attains nearly its stationary value, but then creeps very slowly to its final value. This he called **Viscous Hysteresis.** The phenomenon of viscous hysteresis has no practical interest at present.

When the number of lines of force passing through a coil is changed, an electric pressure is generated, which is in value

$$E = -\frac{ndN}{10^8 dt},$$

where dN and dt are the increments of total induction and of time (see page 39). If a current be flowing in the coil, the work done upon, or by, the lines of force is $CEdt$, and (from above)

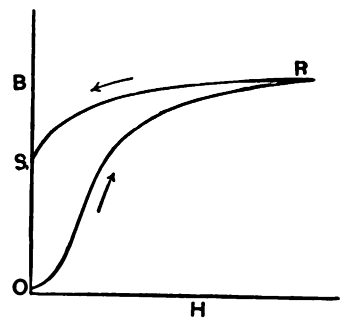


Fig. 27.

$$CEdt = -\frac{nCdN}{10^8};$$

but $N = AB$ and $dN = AdB$, while $nC = \frac{10HL}{4\pi}$,

whence $\quad dW = CEdt = -\frac{AL}{10^7 \times 4\pi} HdB.$

Integrating gives

$$W = CET = -\frac{AL}{10^7 \times 4\pi} \int_{B_2}^{B_1} HdB,$$

and the power in watts expended during the duration of the action is

$$CE = -\frac{AL}{10^7 \times 4\pi T} \int_{B_2}^{B_1} HdB.$$

The value of $\int_{B_2}^{B_1} HdB$ is evidently equal to the area *EFGJ* (Fig. 28).

If the change in the induction be due to a change in the current flowing in the coil, the *C* of the formula must represent the average current, and according to Lenz's law, work must be done by it against a counter electric pressure caused by the increasing induction when the current is increased, and work will be expended by the magnetism when the current is decreased. The work done against a counter electric pressure as the induction is increased, is stored in the magnetic field, and is returned when the induction dies away. In other words, work must *be done upon* the magnetic molecules in order to swing them into line when a test piece is magnetized, while work is *done by* the molecules when they swing back upon the withdrawal of the magnetizing force. As no energy is required to *maintain* a magnetic field when once established, there would be no loss of energy from magnetizing and then demagnetizing a piece of iron, if the curves of magnetization, ascending and descending, were the same. For, $\int_{B_2}^{B_1} HdB$ would be equal and of opposite sign in the two cases. But there is a difference in the two curves, due to hysteresis, and there must be a loss of energy due to

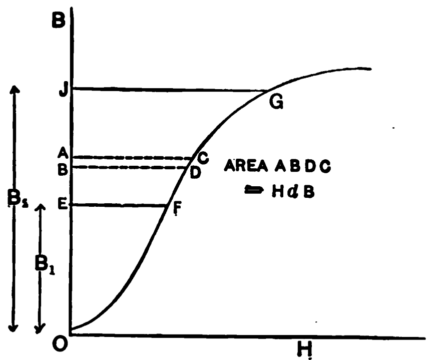


Fig. 28.

the operation, proportional to the area between them.

Or, $$W = -\frac{AL}{10^7 \times 4\pi}\left[\int_{B_2}^{B_1} HdB + \int_{B_3}^{B_2} HdB\right].$$

Hence, $$W = -\frac{AL}{10^7 \times 4\pi}\int_{B_3}^{B_1} HdB.$$

$\int_{B_2}^{B_1} HdB$ is evidently the area ORJ, Fig. 29, $\int_{B_3}^{B_2} HdB$ the area SRJ, and $\int_{B_3}^{B_1} HdB$ is the area ORS.

If a piece of iron be carried through a continuous cycle of magnetization, between equal positive and negative values of H, a cyclic curve of magnetization,

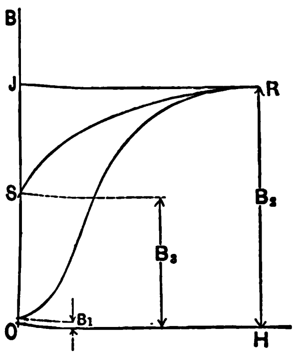


Fig. 29.

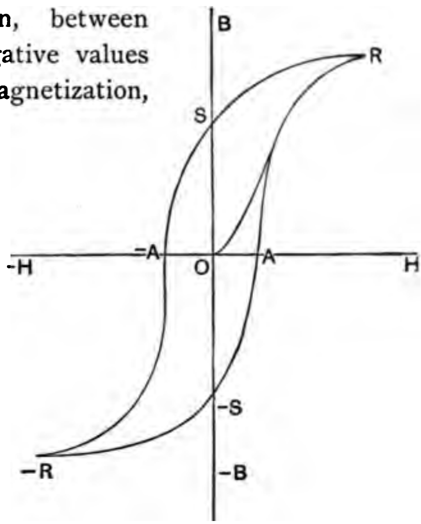


Fig. 30.

similar to Fig. 30, is given. The integral then becomes

$$W = -\frac{AL}{10^7 \times 4\pi}\int_{B}^{-B} HdB,$$

and $$CE = -\frac{AL}{10^7 \times 4\pi \times T}\int_{B}^{-B} HdB.$$

If the cycle be *repeated* continually, as in the core of

an alternating current transformer, or in the revolving armature core of a dynamo, the integral becomes

$$CE = \frac{ALV}{10^7 \times 4\pi \times 60} \int_{B}^{-B} HdB$$

where V is the number of cycles, or revolutions, per minute $\left(= \frac{60}{T}\right)$.

It is to be noted that when the method of reversal (previously discussed) is used for testing, no distinction is made between the ascending and descending curves, and the curve so determined is, presumably, as close to the mean of the two as the physical conditions will permit. In regulating a dynamo, the field magnets are as likely to be brought to any definite induction from a slightly higher one as from a lower one, and the mean curve of magnetization or permeability is therefore of most use in designing field magnets. In dynamo armatures, or alternating current transformers, the iron is subjected to continuous cycles of magnetization, and a careful study of the area included between the ascending and descending curves is necessary, in order that the loss of energy due to the reversals of magnetism in the iron may be reduced to a minimum.

The effect of physical properties and chemical composition upon the area of the cyclic curve of magnetization is very marked. The accompanying figure (31) shows the cyclic curves of magnetization of a soft iron wire; 1st, after annealing, 2d, after hardening by stretching. This is the same wire previously quoted from Ewing, and for which the permeability curves were given. In

neral it may be said, that apparently anything which ıds to reduce the maximum of the permeability ·ve, also tends to increase the area of the cyclic curve

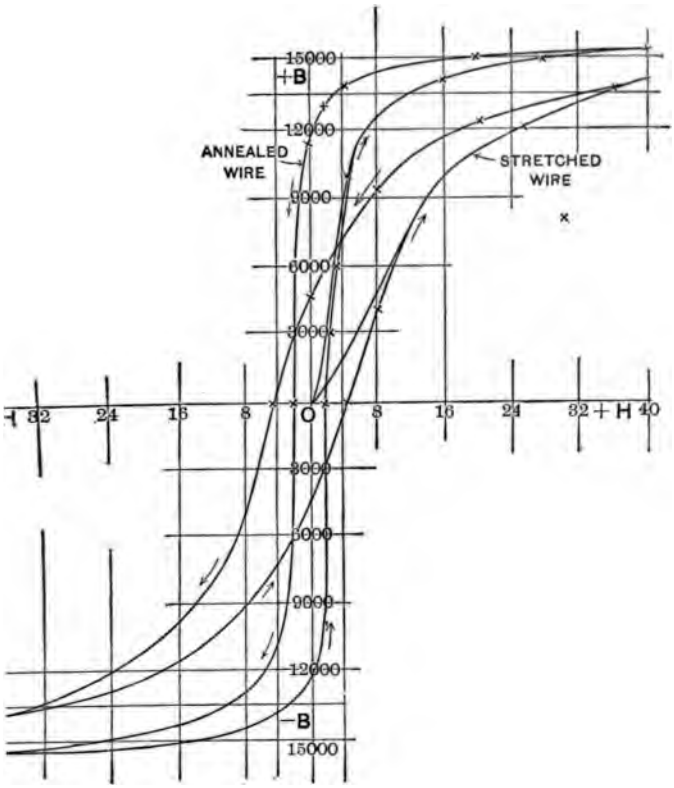


Fig. 31

magnetization, and hence to increase the energy ssipated per cycle. A careful examination of the ations of the two curves will show this to be true.

A much closer experimental study of the effect of chemical impurities, than has yet been made, would be of advantage in pointing towards the best methods of manufacture for mild steel or wrought-iron plates, to be used in alternating current apparatus and in continuous current armature cores.

From the formula for the energy dissipated by reversals, we may say that the power in watts wasted is equal to the area of the cyclic curve multiplied by the volume of the iron (in cubic centimeters) times the number of cycles per minute and divided by 7,500,000,000. This energy must evidently be expended in heating the iron. From an inspection of the curves it is evident that the coercive force is equal to OA, and the retentiveness to OS (Fig. 30); also, that the area of the curve is approximately equal to that of a parallelogram, with a height which is twice the maximum induction, and a base equal to twice the coercive force. Hence, anything that decreases the coercive force will also decrease the energy lost through hysteresis. The coercive force is not a constant for any sample, but increases directly with B. Ewing found the coercive force in very soft iron to be 1.1 when B was 3700, and 1.7 when B was 14,000. The proportional increase is greater in hard iron. Steinmetz found the coercive force of cast iron to be 10 and 15 when B was respectively 6100 and 10,000, while the coercive force of soft machine steel was 9 and 11 when B was 14,000 and 18,800 respectively. Steinmetz has further shown that the continuous waste of energy by hysteresis can be approximately represented by the formula $U=\nu VB^{\frac{8}{5}}$, where U is the power in watts lost per cubic centimeter

of iron, V is the number of cycles per minute, and ν is a constant depending upon the quality of the iron for its numerical value. (See *Transactions American Institute of Electrical Engineers*, Vol. 9.) The value of ν has been found to vary from 33×10^{-13} to 14×10^{-11} for different samples of iron and steel. For the sheet-iron used in a certain commercial transformer, Steinmetz found a value $\nu = 40 \times 10^{-13}$. Since $2^{\frac{8}{5}} = 3$, it is evident from the formula, that doubling the induction trebles the waste due to hysteresis.

The loss due to hysteresis can be calculated from the area of the cyclic curve showing the relation between H and I, as well as from the cyclic curve of magnetization. For, $B = H + 4\pi I$ and $dB = dH + 4\pi dI$. Hence

$$HdB = HdH + 4\pi HdI,$$

but HdH must disappear for a complete cycle, with equal maximum positive and negative values of H. Therefore $HdB = 4\pi HdI$, and by substitution we have

$$W = -\frac{AL}{10^7}\int_I^{-I} HdI.$$

CHAPTER IV.

ESTABLISHMENT OF ELECTRIC PRESSURES.

If two metal rails, as *A* and *B* in Fig. 32, be placed perpendicular to the lines of force in a magnetic field, and a metal slider *C* be moved along them, an electromotive force or electric pressure will be set up in the slider and rails. If the circuit be closed, a current will flow while the slider moves, and it may be measured by

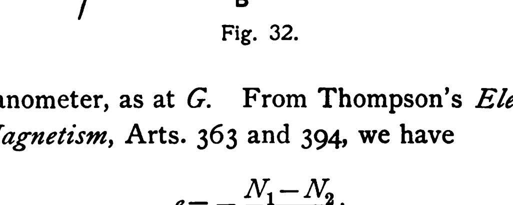


Fig. 32.

a galvanometer, as at *G*. From Thompson's *Electricity and Magnetism*, Arts. 363 and 394, we have

$$e = -\frac{N_1 - N_2}{10^8 t},$$

where e is the electric pressure in volts, and N_1 and N_2 are respectively the number of lines of force passing through or linking the electric circuit (composed of slider, rails, and connections) before the motion begins

and after time t. This is equivalent to $10^8 e = -\frac{dN}{dt}$. With a closed circuit having a resistance of R ohms, the current flowing is $C = \frac{e}{R} = -\frac{dN}{dt} \times \frac{1}{10^8 \times R}$. The negative sign is used to signify that the magnetic field, generated by the induced current, is opposite to the inducing field; or, in other words, the current set up tends to oppose the motion according to Lenz's law. In this case work must be done upon the slider to keep it in motion. If the galvanometer be replaced by a battery or other source of current, giving an electric pressure of e' volts, the slider will tend to move, if not obstructed, at such a velocity that $\frac{dN}{dt} = 10^8 e'$. In this case the slider will do work. If "a" be the perpendicular distance between the rails, and B the induction measured in the plane of the rails, then $dN = Badl$, where l is the length of the rails. Hence $\frac{dN}{dt} = Ba\frac{dl}{dt}$. $\frac{dl}{dt}$ is evidently the velocity of the slider in centimeters per second, and it may be replaced by the letter v; whence $e = -\frac{Bav}{10^8}$. av is evidently the area between the rails swept over by the slider in each second, and the value of e is therefore $\frac{1}{10^8}$ times the *number of lines of force cut through by the slider per second.* Also, if a current be passed through an unobstructed slider, it will tend to move at such a velocity as to cut through a number of lines of force per second *numerically equal to the impressed electric pressure* multiplied by 10^8.

A wire hung from the pole of a magnet, with its ends

dipping in mercury cups at the pole and at the waist of the magnet, as shown in Fig. 33, is practically a continuous slider. When the wire revolves about the magnet, the electric pressure generated is

$$e = -\frac{dN}{10^8 dt} = -\frac{NV}{10^8} = -\frac{4\pi m V}{10^8},$$

where V is the velocity in revolutions per second, and $4\pi m$ is evidently equal to N, the total number of lines of force emanating from the pole.

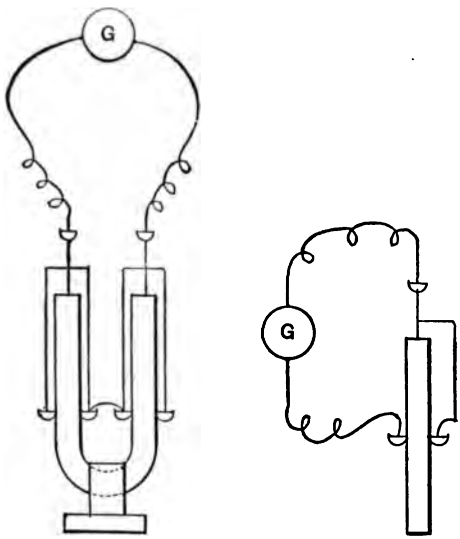


Fig. 33.

A disc rotating in a uniform magnetic field, as shown in Fig. 34, is another type of continuous slider. In this case, N is equal to $\pi r^2 B$, and the electric pressure generated by each radius is $e = -\frac{dN}{10^8 dt} = -\frac{\pi r^2 B V}{10^8}$, where V is the velocity in revolutions per second. The pres-

sure developed may be measured by means of a galvanometer connected between brushes placed on the shaft and on the periphery of the disc. If the field be not uniform in this case, as when the disc is placed between the poles of a horseshoe magnet, the value of $\frac{dN}{dt}$ is different at different points; hence currents will circulate in the body of the disc, and the external pressure will depend upon the position of the brush on the disc's periphery, and upon the distribution of the field. The parasitic currents which circulate in the body of the disc are called **Foucault or Eddy Currents.**

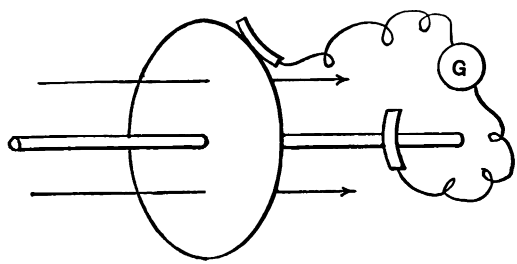


Fig 34.

If a coil of n_1 turns of wire be moved parallel to itself from a point in a magnetic field to a point of different strength, the electric pressure developed is, as before, $e = -\frac{n_1}{10^8} \times \frac{dN}{dt}$, where $\frac{dN}{dt}$ is the rate of change in the number of lines of force linking the coil. $n_1 \frac{dN}{dt}$ is readily seen to be equal to the algebraic summation of the *rate* at which lines of force are cut by the wires of the coil. If the coil be rotated as well as translated, the electric pressure at any moment is, as before, $e = -\frac{n_1}{10^8} \times \frac{dN}{dt}$;

but here, $N = BA \cos\alpha$, where A is the mean area of the coil and α is the angle which a normal to the plane of the coil makes with the direction of the lines of force. dN is therefore equal to $-BA \sin\alpha\, d\alpha$, and

$$e = -\frac{n_1}{10^8} \times \frac{dN}{dt}$$

becomes

$$e = \frac{1}{10^8} BAn_1 \sin\alpha \frac{d\alpha}{dt}.$$

$\frac{d\alpha}{dt}$ is evidently equal to the instantaneous angular velocity of rotation, which may be represented by Greek ω. Hence $e = \frac{1}{10^8} n_1 BA\omega \sin\alpha$. Since $BA \sin\alpha$ is proportional to the change in the number of lines of force passing through the coil, and ω is the angular velocity, the last formula shows the instantaneous electric pressure to be as before, numerically equal to the algebraic summation of the rate at which lines of force are cut by the wires of the coil. If T be the time of a complete revolution in seconds, $\omega = \frac{2\pi}{T}$. If an angular displacement α be made in time t, then $\alpha = \frac{2\pi t}{T}$, and hence

$$e = \frac{2\pi}{10^8 T} BAn_1 \sin\frac{2\pi t}{T} = \frac{2\pi n_1 N}{10^8 T} \sin\frac{2\pi t}{T}.$$

If V represent the number of revolutions made by the coil per minute, $T = \frac{60}{V}$, and the formula for the electric pressure becomes

$$e = \frac{2\pi n_1 NV}{10^8 \times 60} \sin\frac{2\pi t}{T} = \frac{2\pi n_1 NV}{10^8 \times 60} \sin\alpha.$$

This is the equation of a sinusoid, or sine curve. The instantaneous values of e at different parts of the

revolution are proportional to the ordinates of the curve, the base of which is equal to T (Fig. 35). The formula shows that the maximum positive ordinate is at 90° or $\frac{1}{4}$ revolution, and the maximum negative ordinate is at 270° or $\frac{3}{4}$ revolution, while the curve cuts the x axis at 0°, 180°, and 360°. The instantaneous pressures in the revolving coil are therefore zero when $t=0$, $\frac{1}{2}T$, and T, and are a maximum when $t=\frac{1}{4}T$ and $\frac{3}{4}T$. The average

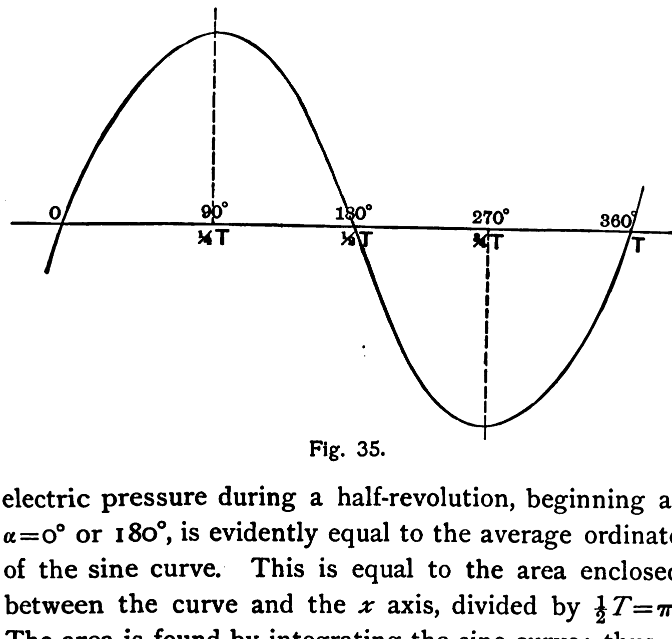


Fig. 35.

electric pressure during a half-revolution, beginning at $\alpha=0°$ or 180°, is evidently equal to the average ordinate of the sine curve. This is equal to the area enclosed between the curve and the x axis, divided by $\frac{1}{2}T=\pi$. The area is found by integrating the sine curve; thus

$$A=2\frac{2\pi n_1 NV}{10^8\times 60}\int_0^{\frac{1}{2}\pi}\sin\alpha d\alpha=2\frac{2\pi n_1 NV}{10^8\times 60}\Big[-\cos\alpha\Big]_0^{\frac{1}{2}\pi}$$

$$=\frac{4\pi n_1 NV}{10^8\times 60}. \quad \text{Whence } E=\frac{4\pi n_1 NV}{10^8\times 60}\div\pi=\frac{4 n_1 NV}{10^8\times 60}.$$

The area of the curve during the second half-revolution of the coil has the same numerical value as in the first half, but it lies below the x axis. The average electric pressure during the second half-revolution is, therefore, numerically equal to that of the first half, but is opposite in direction. The value of the electric pressure that is developed in a moving coil can also be shown by considering the coil in projection, and applying the laws of the slider.

The instantaneous current in the coil, if the circuit is closed at any moment, is $c=\frac{e}{R}=\frac{2\pi n_1 NV}{10^8\times 60\times R}\sin\alpha$, and the number of coulombs of electricity transferred is $q=\frac{2\pi n_1 NVt}{10^8\times 60\times R}\sin\alpha$. Integrating as before gives for the half-revolution $Q=\frac{4n_1 NVT}{10^8\times 60\times R}$. Whence the average current is

$$C=\frac{Q}{T}=\frac{4n_1 NV}{10^8\times 60\times R}.$$

The maximum values of electric pressure and current are given when $\alpha=90°$, at which point $e_{max}=\frac{2\pi n_1 NV}{10^8\times 60}$, and $c_{max}=\frac{2\pi n_1 NV}{10^8\times 60\times R}$. The ratios $\frac{E}{e_{max}}$ and $\frac{C}{c_{max}}$ are then equal to $\frac{2}{\pi}=.637$.

The power expended by a sinusoidal current, for instance in heating a wire, is not simply proportional to the square of the mean current, as in the case of continuous currents, but is proportional to the summation of the square of each instantaneous current (*i.e.* Σc^2).

But $$\Sigma c^2 R = \frac{2R}{\pi} \frac{4\pi^2 n_1^2 N^2 V^2}{10^{16} \times 3600 \times R^2} \int_0^{\frac{1}{2}\pi} \sin^2 \alpha d\alpha$$

$$= \frac{2\pi^2 n_1^2 N^2 V^2}{10^{16} \times 3600 \times R},$$

and $$C^2 R = \frac{16 n_1^2 N^2 V^2}{10^{16} \times 3600 \times R},$$

whence $$\frac{C^2 R}{\Sigma c^2 R} = \frac{16}{2\pi^2} = .81.$$

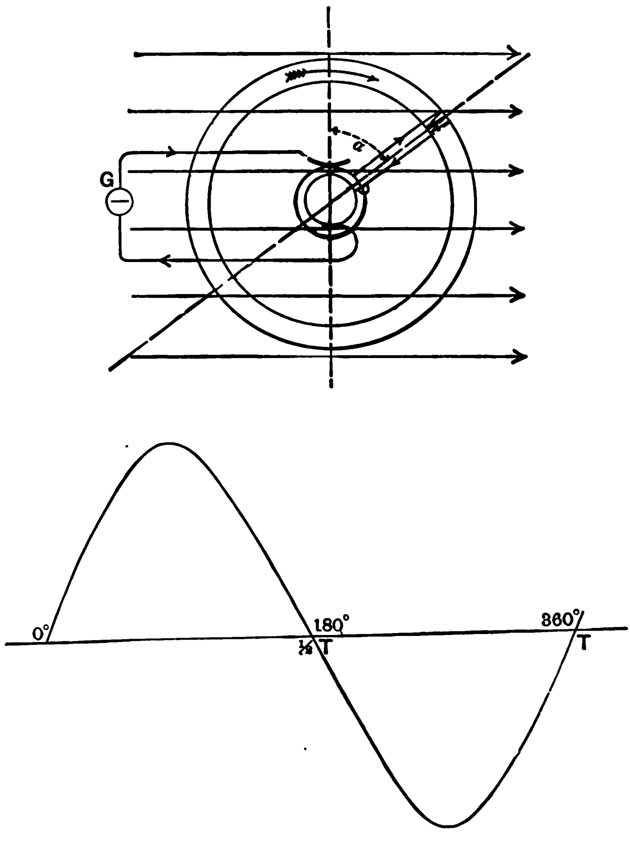
The heating effect of a sinusoidal current of mean value C amperes is thus shown to be greater than that of a continuous current of C amperes in the ratio of 1 : .81. The difference is caused by the undue weight of the larger values of c in the expression $\Sigma c^2 R$. $\sqrt{\Sigma c^2}$ is sometimes called the **Effective Current,** and the ratio of the mean current to it is $\frac{C}{\sqrt{\Sigma c^2}} = .9$. The ratio of the effective current to the maximum current is .707.

A sinusoidal current produced in the manner thus far assumed is called an **Alternating Current** (Fig. 36). Such a current can be **Commutated** or **Rectified** by proper devices, so that its direction will remain constant (Fig. 37), but its magnitude will continue to vary according to the sine law. For the rectified current the mean pressure will be

$$E = \frac{4 n_1 N V}{10^8 \times 60},$$

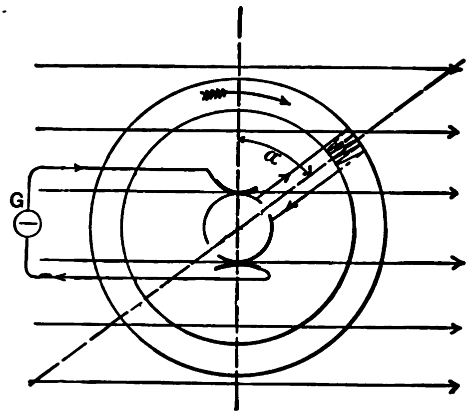
as before, and continuous in direction. The arrangement of the commutator and collecting brushes is evident.

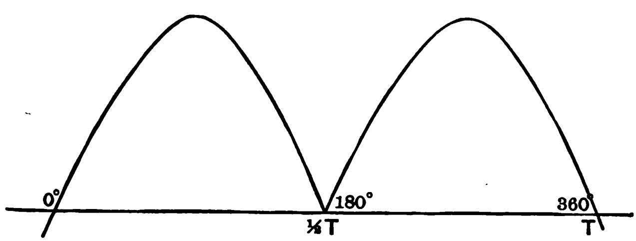
If two coils be placed symmetrically with reference to the axis of rotation, and connected to the commutator



ALTERNATING CURRENT.

Fig. 36.





COMMUTATED OR RECTIFIED CURRENT.

Fig. 37.

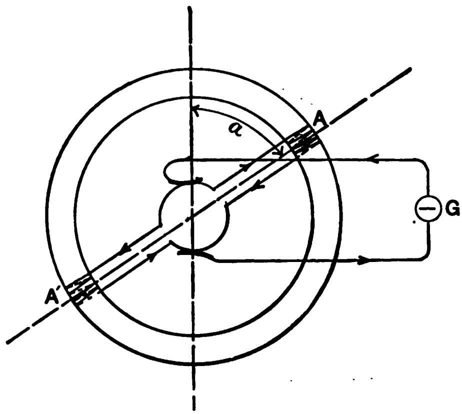


Fig. 38.

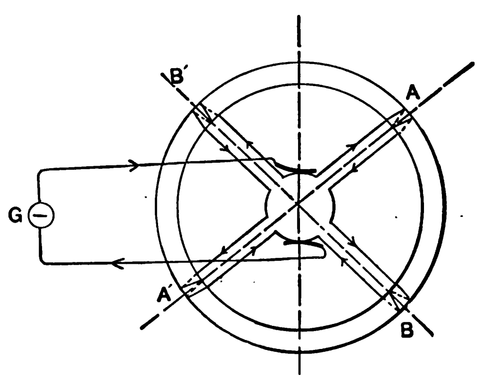


Fig. 39.

in parallel, as shown in Fig. 38, the electric pressure in the external circuit remains the same as for one coil, but each coil furnishes *half of the current flowing.* The value of E remains as before but the formula may be changed in form by replacing n_1, the number of turns on one coil, by n_2, the total turns of the two coils $(=2n_1)$, and replacing N by $N_2=2N=$ sum of lines of force passing through the coils. Then

$$E=\frac{n_2 N_2 V}{10^8 \times 60}.$$

If an additional pair of coils BB' be placed at 90° from the first pair AA', and the commutator be properly arranged, as shown in Fig. 39, the external pressure at any instant is the superposed effect of two coils, with a difference of phase (position) of 90°. The instantaneous electric pressure is

$$e=\frac{\pi n_2 N_2 V}{2 \times 10^8 \times 60}[\sin \alpha+\sin (\alpha+90^\circ)],$$

and the average

$$E=\frac{2 n_2 N_2 V}{10^8 \times 60}.$$

If, finally, "a" pairs of coils be uniformly spaced around a ring, and connected to a proper commutator of $2a$ parts, the instantaneous pressure at the brushes is the summation of the pressure due to "a" coils, and

$$e=\frac{\pi n_2 N_2 V}{2 \times 10^8 \times 60}\left[\sin \alpha+\sin\left(\alpha+\frac{\pi}{a}\right)+\cdots+\sin\left(\alpha+\frac{a-1}{a}\pi\right)\right],$$

whence $$E=\frac{an_2N_2V}{10^8\times 60}\int_0^{\frac{1}{a}\pi}\left[\sin\alpha+\sin\left(\alpha+\frac{\pi}{a}\right)+\cdots\right.$$

$$\left.+\sin\left(\alpha+\frac{a-1}{a}\pi\right)\right]d\alpha$$

$$=\frac{an_2N_2V}{10^8\times 60}.*$$

$an_2(=2an_1)$ is equal to the total number of turns on the ring, and it also is equal to the total number of

RESULTANT PRESSURE FOR 2 COILS.

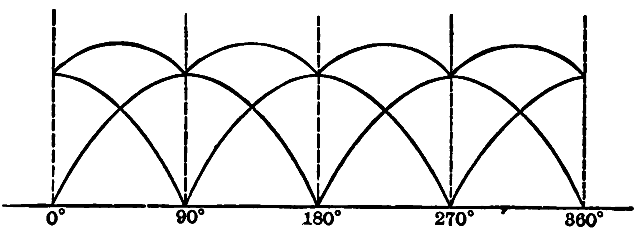


Fig. 40.

$$*\int_0^{\frac{1}{a}\pi}\left[\sin\alpha+\sin\left(\alpha+\frac{\pi}{a}\right)+\sin\left(\alpha+\frac{2\pi}{a}\right)+\cdots+\sin\left(\alpha+\frac{(a-1)\pi}{a}\right]\right)d\alpha$$

$$=-\left[\cos\alpha+\cos\left(\alpha+\frac{\pi}{a}\right)+\cos\left(\alpha+\frac{2\pi}{a}\right)+\cdots+\cos\left(\alpha+\frac{(a-1)\pi}{a}\right]_0^{\frac{1}{a}\pi}=1.$$

The upper limit gives

$$-\left(\cos\frac{\pi}{2a}+\cos\frac{3\pi}{2a}+\cos\frac{5\pi}{2a}\right)+\cdots+\cos\left(\pi-\frac{\pi}{2a}\right)=0.$$

The lower limit gives

$$-\left(1+\cos\frac{\pi}{a}+\cos\frac{2\pi}{a}\right)+\cdots+\cos\left(\pi-\frac{\pi}{a}\right)=-1.$$

The integration therefore equals unity as given above, and the average electric pressure is

$$E=\frac{an_2N_2V}{10^8\times 60}=\frac{nN_2V}{10^8\times 60}.$$

Compare Todhunter's Trigonometry, page 241.

external conductors on the ring. Writing n in place of an_2 gives $E=\frac{nN_2V}{10^8\times 60}$. While the coils revolve through the angle $\alpha=\frac{\pi}{a}$, starting at $\alpha=-\frac{1}{2}\frac{\pi}{a}$ and ending at $\alpha=\frac{1}{2}\frac{\pi}{a}$, the electric pressure passes from its highest value to its lowest value and returns again to its highest value; that is, the pressure is a maximum when $\alpha=\pm\frac{1}{2}\frac{\pi}{a}$ and a minimum when $\alpha=0$. The former is when the brush rests at the centre of one of the commutator segments, and the latter when the brush bridges the insulation between two segments.

This angular revolution equals the angular width of a commutator division, and the maximum and minimum instantaneous electric pressures are respectively,

$$e_{\max}=\frac{\pi}{2}\,\frac{n_2N_2V}{10^8\times 60}\operatorname{cosec}\frac{1}{2}\frac{\pi}{a},$$

$$e_{\min}=\frac{\pi}{2}\,\frac{n_2N_2V}{10^8\times 60}\cot\frac{1}{2}\frac{\pi}{a};\,*$$

* For, when $\alpha=\frac{1}{2}\frac{\pi}{a}$,

$$\sin\alpha+\sin\left(\alpha+\frac{\pi}{a}\right)+\sin\left(\alpha+\frac{2\pi}{a}\right)+\cdots+\sin\left(\alpha+\frac{(a-1)\pi}{a}\right)$$

becomes

$$\sin\frac{1}{2}\frac{\pi}{a}+\sin\frac{3\pi}{2a}+\sin\frac{5\pi}{2a}+\cdots+\sin\left(\pi-\frac{\pi}{2a}\right)=\frac{\cos 0^\circ-\cos\pi}{2\sin\frac{\pi}{2a}}=\operatorname{cosec}\frac{\pi}{2a}.$$

When $\alpha=0$,

$$\sin\alpha+\sin\left(\alpha+\frac{\pi}{a}\right)+\sin\left(\alpha+\frac{2\pi}{a}\right)+\cdots+\sin\left(\alpha+\frac{(a-1)\pi}{a}\right)$$

becomes

$$\sin\frac{\pi}{a}+\sin\frac{2\pi}{a}+\cdots+\sin\left(\pi-\frac{\pi}{a}\right)=\frac{\cos\frac{\pi}{2a}-\cos\left(\pi-\frac{\pi}{2a}\right)}{2\sin\frac{\pi}{2a}}=\cot\frac{\pi}{2a}.$$

Compare Todhunter's *Trigonometry*, page 241.

The minimum pressure occurs at the instant that one pair of coils is in the position at which it does not cut lines of force. The ratio of the maximum to the minimum pressure is equal to $\dfrac{1}{\cos\frac{1}{2}\frac{\pi}{a}}$, which becomes very nearly equal to unity as "a" increases. The fluctuation of pressure therefore becomes very small as the coils increase in number, and the continuous pressure throughout the revolution becomes virtually equal to $\dfrac{nN_2V}{10^8\times 60}$.

When the coils revolve through the angle $\dfrac{\pi}{a}$, beginning at $90°-\frac{1}{2}\frac{\pi}{a}$ and ending at $90°+\frac{1}{2}\frac{\pi}{a}$, the definite integral

$$\int_{90°+\frac{1}{2}\frac{\pi}{a}}^{90°-\frac{1}{2}\frac{\pi}{a}}\left[\sin\alpha+\sin\left(\alpha+\frac{\pi}{a}\right)+\sin\left(\alpha+\frac{2\pi}{a}\right)+\cdots\right.$$
$$\left.+\sin\left(\alpha+\frac{a-1}{a}\pi\right)\right]d\alpha$$

reduces to zero, and hence there is no difference of pressure between the points in the coils at $\alpha=90°$ and $\alpha=270°$. The maximum difference of pressure occurs between the points $\alpha=0$ and $\alpha=180°$; hence the collecting brushes of dynamos should touch these points on the commutator, if there are no disturbing influences.

The total fluctuation while the coils revolve through the angle from $\alpha=-\frac{1}{2}\frac{\pi}{a}$ to $\alpha=\frac{1}{2}\frac{\pi}{a}$ is evidently $e_{max}-e_{min}$ $=\left(1-\cos\frac{1}{2}\frac{\pi}{a}\right)\times$ a constant. The values of $1-\cos\frac{1}{2}\frac{\pi}{a}$ for various values of "a" from 1 to 180 are plotted in the accompanying curve, Fig. 41.

The percentage of the average pressure which is represented by the fluctuation decreases in the same ratio as the number of coils increases.

The effect of self-induction when a current flows has been neglected in this discussion, and its consideration

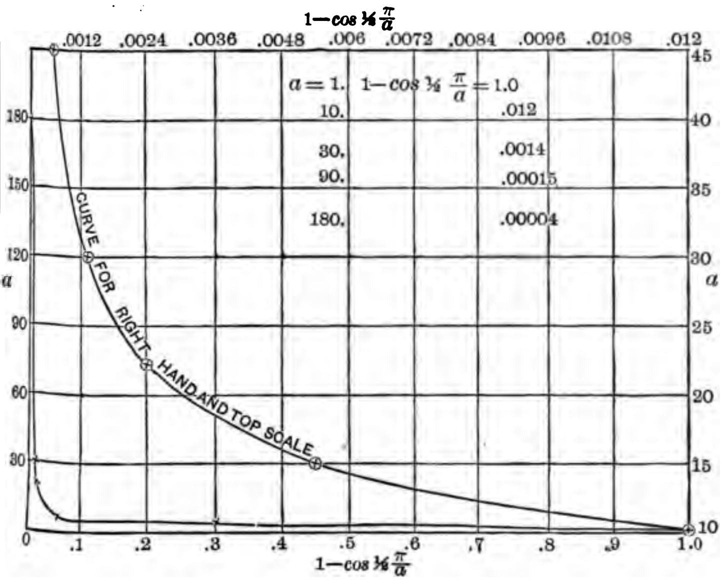


Fig. 41.

is reserved for a later page. It is sufficient to say here that its general tendency is to decrease the fluctuations, and also to decrease slightly the mean electric pressure developed. The fluctuation curve shows that when $2a$ (total number of coils) is from 30 to 80, the fluctuation can be practically neglected, and, as already stated, self-induction serves to smooth out the variations still

farther. It is therefore usual to treat the pressure developed in a dynamo armature of from 30 coils upwards, as a *constant pressure.* A current, constant in direction and value (due to a constant pressure), is usually called a **Continuous Current**. A current constant in direction, but not necessarily so in value, is often called a **Direct Current**.

There is a modification of the general case of a set of coils revolving about a common axis, which requires

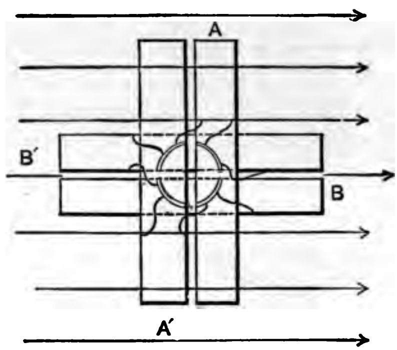


Fig. 42.

attention. If the coils of each pair be moved so as to be side by side and with their centre in the axis of revolution, it is evident that double the preceding formulas represents the electric pressure, and

$$E = 2\,\frac{n\,N_2 V}{10^8 \times 60}.$$

For the total number of lines of force (equal to $N_2 \cos\alpha$) now pass through both coils of each pair at any moment, giving the effect due to N_2 lines of force for

each coil, while in the general case the number of lines of force at any moment passing through each coil of a pair is $\frac{1}{2} N_2 \cos\alpha$, giving the effect due to $\frac{1}{2} N_2$ lines of force for each coil.

The general case is evidently a ring wound with $2a$ coils uniformly spaced, and arranged to revolve about its true axis. The modified case consists of a cylinder upon which $2a$ coils are wound longitudinally, and which is arranged to revolve about its axis. In the case of the ring, the number of wires (or conductors) upon its outer surface is evidently equal to n, while in the case of the cylinder the number of conductors on the surface is $2n$. Whence, writing S for the number of conductors on the surface in either case, and N_a for the number of lines of force passing through the armature ($=N_2$), the general formula for the electric pressure becomes

$$E = \frac{S N_a V}{10^8 \times 60}.$$

This can be formulated as a rule thus:

The commutated electric pressure developed in any armature, composed of a set of coils wound on a ring or drum and revolved about their common axis, is equal to the *number of lines of force cut per second*, multiplied by the *number of external conductors on the ring or drum*, and divided by 10^8.

It can now be seen that the development of electric pressure, by the movement of wires in a magnetic field, can be classified under three divisions:

1. Where a wire cuts the lines of force by moving across them; as in the case of a slider or of a wire moving around a magnet pole.

2. Where a coil, or set of coils, is moved parallel to itself, or nearly so, between points of different strength in a magnetic field.

3. Where a coil, or set of coils, is wound on a ring or drum and given a rotary motion in a fixed magnetic field.

The pressure developed by the first division is often spoken of as caused by **Unipolar Induction.** This term is erroneous, if strictly interpreted, as induction due to one pole (unipolar) cannot exist. The term can be satisfactorily applied, however, to distinguish the first division if its meaning be not literally construed. The first division has been shown to cover the fundamental case, and the second and third divisions can be considered as special cases. On account of the commercial importance of the latter, they will be given separate treatment. It is to be remembered that electrical pressures produced under the conditions defined as belonging to the first division, are rigorously continuous, provided V be inversely proportional to B, while the practical conditions of the second and third divisions produce either alternating or rectified pressures. The rectified pressures approximate more or less closely to rigorous continuity only as "a" approaches a large numerical value.

In commercial machines intended to produce electrical pressures by induction, *i.e.* **Dynamos,** the parts of the machines in which the pressures are produced are called **Armatures.** In considering the requirements to be met in dynamo armatures, it is well to make a classification upon less scientific but more practical lines

than the division already made. Here again the classes are three in number, but the division is based upon the *results produced by* the machines, instead of the *manner in which the results are produced.* The three classes of armatures are:

1. Rigorously continuous current armatures. These belong in the first division, as already enumerated.

2. Alternating current armatures. A majority of these belong to the third division, and the remainder to the second division, as already enumerated.

3. Ordinary continuous current armatures. These belong for the most part to the third division of the previous classification.

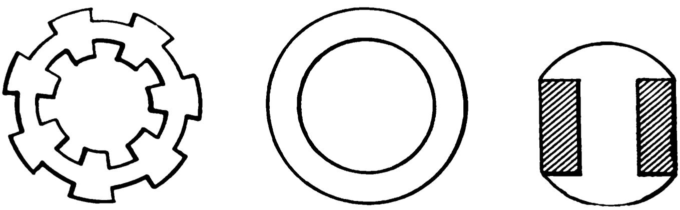
The second class given here has no place in the present volume, and the first class will receive only a hasty consideration on account of its small commercial usefulness. The third class covers nearly all armatures of commercial continuous current machines. This class, or the rotating coil armatures, must be again divided, for convenience, into three types:

1. *Gramme* or *ring* armatures.
2. *Siemens* or *drum* armatures.
3. *Open coil* armatures.

The Gramme and Siemens armatures are often called **Closed Coil** armatures to distinguish them from the third type.

When the ring armature carries its conductors in grooves, it is often called a **Pacinotti** armature, after its first inventor, who described it in 1864. The drum type is often called after **Hefner von Alteneck**, who

designed the first one with multiple coils in 1873. The ring armature of Pacinotti was reinvented and first put into commercial form by Gramme in 1871. In Gramme's form, the ring was not toothed and the coils were wound uniformly over its surface. The first practical armature approaching the drum type in form had a single coil and a two-part commutator. The core was



PACINOTTI CORE.
Coils wound in groves.

GRAMME CORE.
Coil wound uniformly.

CROSS-SECTION OF SHUTTLE ARMATURE.
Windings shown by hatchings.

Fig. 43.

a cylinder with two longitudinal grooves, in which the windings were laid. This was invented by Werner Siemens before 1856. It is often called the **Shuttle Wound** or **H** armature, on account of its form.*

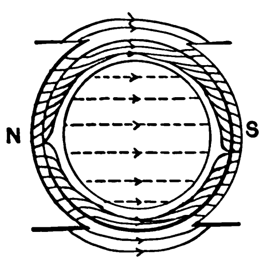


Fig. 44.

Since the armatures of all commercial machines of the class under consideration have iron cores, the number of lines of force useful in producing pressure is the total number cut by the external conductors, both in the ring and drum armatures. This is evident in the case of

* See THOMPSON'S *Dynamo-Electric Machinery*, Chap. 2; URQUHART'S *Dynamo Construction*, Introduction; KAPP'S *Electric Transmission of Energy*, Chap. 2; etc.

the drum armature, and requires no further comment. In the ring, the iron core provides a path of low reluctance for the lines of force, and therefore few lines pass directly across the central space. These few can be considered negligible, or a correction made for them, as the case may demand, when N is taken as the total number of lines cut by the external conductors (Fig. 44).

Armature Winding for Two-pole Dynamos.

A. **Gramme Type.** As shown in the preceding discussion, the Gramme armature may be looked upon essentially as a series of $2a$ coils uniformly spaced around an iron ring, upon which they are wound. Each coil has its terminals connected to adjoining segments

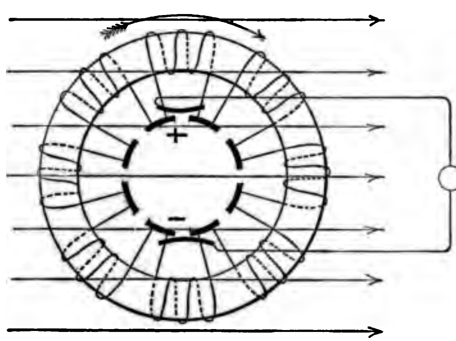


Fig. 45.

or divisions of a commutator, containing $2a$ divisions. This arrangement gives a continuous closed electric circuit in the windings around the ring, exactly as would be given by a continuous helix (Fig. 45). If brushes be

placed upon the commutator at opposite ends of a diameter and a current is passed from one to the other, it must divide between two paths composed of the two halves of the armature. In a discussion of the action of the armature conductors as they cut lines of force, and the resulting electric pressure, drawings are very convenient. For convenience of reference to the figures given hereafter, it is necessary to make a conventional arrangement for the purpose of denoting the direction of currents in conductors shown in cross-section. Individual conductors to be seen in cross-section will be denoted by small circles. When such wires carry a current, the direction of which is to be shown, they will be marked with a + or – sign, respectively, according as the current flows out of the plane of the paper towards the reader, or into the plane of the paper from the reader. With the assistance of this convention, and the well-known modification of Ampère's rule,* the figures show the principles of Gramme armature windings very simply.

Two-pole machines only will be discussed at present, the modifications required for **Multipolar** machines being taken up later. Instead of a series of insulated coils on a ring, the Gramme armature may be looked upon as a *continuous helix* (as already stated) of copper wire wound on an iron core, the whole being arranged with

* If a conductor, in which a man is lying so as to look "down" along the lines of force, tends to move towards his left, the man is swimming with the current. In a dynamo armature, a current is caused to flow when the conductors are moved against this tendency (Lenz's law); that is, when the conductors are moved towards the man's right hand. In a motor, the armature conductors are caused to move towards the left by the action of the magnetic field when a current flows.

proper insulation, and mechanical devices for rotation between magnet poles and for collection of the current. For the latter purpose the brushes are usually made of thin copper sheets or wires. Each segment or division of the commutator is connected to a point on the helix, and the winding is thus divided into $2a$ equal parts. The helix can be either right-handed or left-handed.

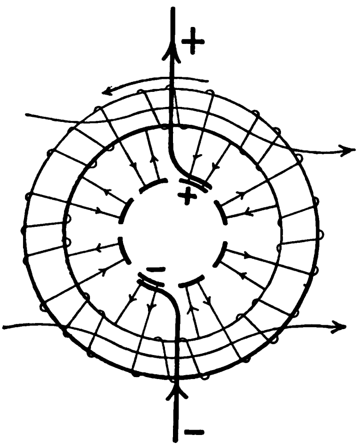


Fig. 46.

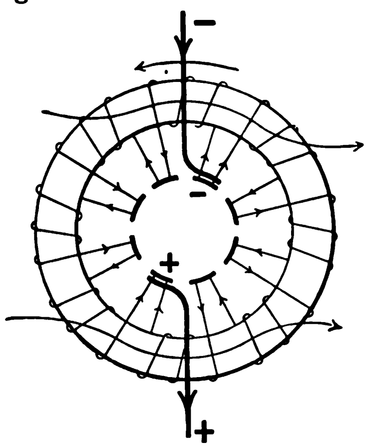


Fig. 47.

When revolved towards the right hand in a right-handed field, as looked upon from the front or commutator end of the armature, the lower brush will be +, and the upper −, in a ring which is wound right-handed. In a left-handed ring, under the same conditions, the upper brush will be positive. This is shown in Figs. 46 and 47 for left-handed rotation in a right-handed field, in which case the polarities are reversed.

In the winding of a Gramme armature, the wire is sometimes actually wound on continuously, taps being attached at proper points for connection to the commu-

tator. But it is usual to wind the coils separately side by side. In this case the ends are led to the respective commutator divisions. Thus the beginning of coil 1 and the end of coil *2 a* goes to the first division, the end of coil 1 and the beginning of coil 2 to the second division, etc., making the effect of a continuous helix, as before.

Economy of construction usually does not allow an armature to be sufficiently large to generate commercial pressures with a single layer of wire, unless the capacity of the machine is great, or the desired pressure is small. The coils, therefore, are usually wound with several layers of wire, one on top of the other. In arc-light dynamos or other machines built for high pressures, the depth of winding sometimes reaches ten or twelve layers on the outer surface of the ring.

As the length of the inner circumference of the ring is less than that of the outer, it is evident that the depth must be greater on the inner surface. Thus a case might occur where each coil contained six turns placed upon the outer surface in two layers of three conductors, while it would be necessary to put them in three layers of two each on the inner side of the ring, as in Fig. 48.

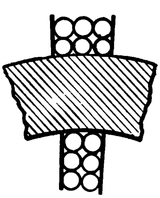


Fig. 48.

When a Gramme armature is of great radial depth as compared with its length, it is usually called a **Disc Armature**. In such armatures the pole pieces are so placed that the conductors on the sides of the disc cut lines of force, instead of the conductors upon the circumference, as in the ordinary ring armatures.

B. **Siemens Type.** The winding of Siemens armatures is not as simple as is that of Gramme armatures. As in the latter, the coils of the Siemens armatures are

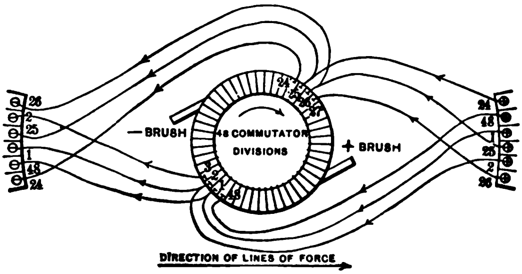


Fig. 49.

connected to the commutator so that the winding is equivalent to a winding made with a *single wire.* The winding may also be either right-handed or left-handed, and the position of the positive brush is determined as in a Gramme armature.

The end of coil 2*a* and the beginning of coil 1 are connected to the first division of the commutator; the

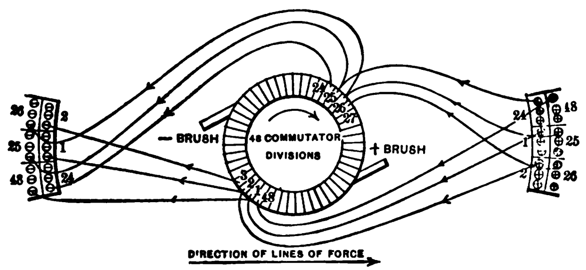


Fig. 49*a*.

end of coil 2 and the beginning of coil 3 to the third division, etc. Figures 49 and 50 show a single turn

winding both left-hand and right-hand, and Figs. 49*a* and 50*a* show a three turn winding. In the single turn winding coils between whose number there is a differ-

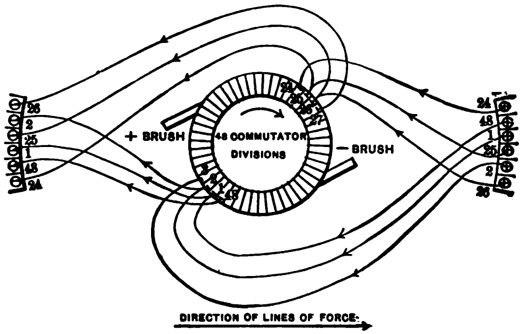


Fig. 50.

ence of "*a*" or of $a \pm 1$, lie side by side; the value of the difference depending on the winding, and varying at different parts of the circumference of an armature.

If each coil consists of several turns, they may all lie side by side provided the circumference of the armature

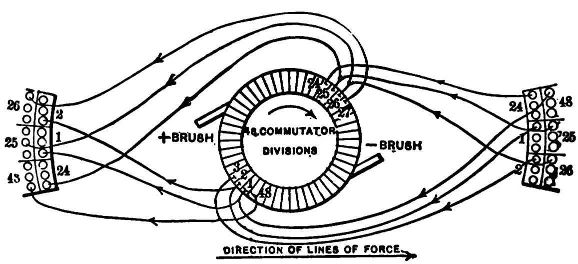
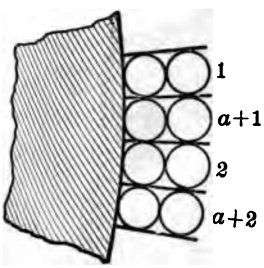


Fig. 50*a*.

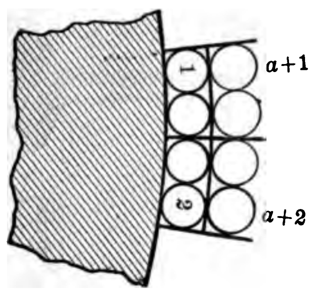
be sufficiently large. This is not usually the case, except in machines of large size or specially low pressures. The windings must therefore usually be made

in more than one layer, and the layers may be arranged either in *vertical* or *horizontal* order (Figs. 51 and 52). In the former case the coils are wound side by side in the same way as the single layer coils, and the order of winding may be coil number 1, coil number $a+1$, coil number 2, coil number $a+2$, etc. In the latter case the first half of the coils are wound in numerical order around the drum, and in such a number of layers as will exactly fill the circumference of the drum. The



VERTICAL WINDING.

Fig. 51.



HORIZONTAL WINDING.

Fig. 52.

second half of the coils are wound in numerical order over those already wound, and in an equal number of layers. In horizontal windings, the number of an outer coil is usually "a" plus the number of the coil under it. The figures show vertical and horizontal windings for two-turn coils, where the circumference accommodates one half the total number of conductors, and the winding is therefore in two layers. They make evident the fitness of the terms "vertical" and "horizontal."

In certain types of windings, the coils are wound upon forming frames, and after a thorough insulating

process, they are placed on the core. Figure 53 shows a single coil and a complete armature of the Eickemeyer type, in which the coils are made on exactly similar formers. Sometimes straight coils wound on formers are used, but the formers must then be of different sizes.

Before the windings are put on an armature core it is usual to insulate it with two or more layers of shellacked canvas, oiled paper, rubber tape, or similar insulating material, and the whole usually receives a good coat of

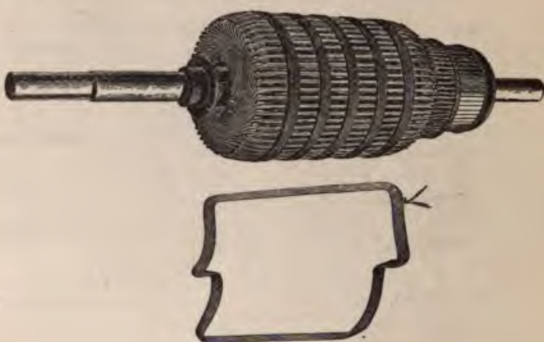


Fig. 53.

Japan or shellac varnish. Where coils cross each other at the ends of drum armatures, or lie alongside of each other on drum or ring armatures, they are separated by thin vulcanized fibre, oiled paper, canvas, mica, etc. Gramme armatures can be more readily insulated than Siemens armatures, as the coils lie smoothly side by side, and thus can be effectually separated. In Siemens armatures, the coils cross over each other in passing across the ends, or heads, and satisfactory insulation becomes a more difficult matter. By liberal use of canvas, oiled

paper, mica, and japan, the insulation can be made satisfactory except for high pressures. The horizontal form of winding causes less difficulty on account of coils crossing on the heads, than does the vertical winding, but it introduces a lack of equality in the coils, which is a compensating disadvantage. The wire that is used in winding armatures is usually covered with two layers of raw cotton thread, the layers being wrapped in opposite directions. It is thoroughly impregnated with japan or shellac when placed on the armature. In some types of windings bare wires are used. In this case the wires are separated by strips of fibre, or similar insulation, and the core insulation is depended upon to prevent grounding. In other cases, bare copper strips compose the windings of toothed armature cores, and the insulation is effected by a sheathing of fibre or shellacked canvas, which is placed around each strip.

In order that the production of *eddy currents* may be avoided in the iron cores of armatures, the cores must be subdivided by planes of subdivision perpendicular to the axis of rotation and parallel to the lines of force. Hence it is usual to build up the cores from discs of annealed sheet iron insulated from each other. The cores of Gramme armatures are sometimes made of annealed iron wire or tape coiled on a spider.

On page 31 is given a relation between the external surface of a coil and the C^2R loss which can safely be allowed in the coil. In that case 75° F. was assumed as the maximum allowable rise of temperature, and in armatures it is not permissible to allow a rise of temperature of much more than about 40° C., or 75° F., above

that of the surrounding air. The air of an engine room on a hot summer's evening sometimes exceeds 100° F., which brings an armature, in which the internal heating causes a rise of 75°, to a greater temperature than 175° F. This cannot be much exceeded without affecting the insulation prejudicially. When an armature is exposed to high temperature, the varnish or japan is softened and runs out, and the canvas and oiled paper become slowly charred. This finally results in the current breaking through the damaged insulation, and consequent injury to, or destruction of, the armature. Excessive heating also injures an armature and its commutator, through the warping effect of the expansion and contraction. The machines of many manufacturers are permitted to heat to a limit much higher than that here given, some even reaching 80° C., but in the best practice the temperature limit is being reduced.

In the case of a fixed coil, as already cited, it is found that the continuous loss of one half a watt in its windings for each square inch of surface will ordinarily cause a rise in temperature of about 75° F. A revolving armature acts to some extent as a fan, causing a current of air to circulate about it, with the effect of carrying the heat off from the surface quite rapidly. It is found that under average conditions of construction and operation, from 2 to $2\frac{1}{4}$ watts can be dissipated (by radiation and convection) from each square inch of external surface of an armature, without causing its temperature to exceed the temperature of the surrounding air by more than about 40° C. The arrangements made for *internal ventilation* of the armature vary the allowable loss per square inch

of surface between wide limits, and the constant given here can only be considered as approximately covering average conditions, where little ventilation is attempted.

The heat produced in an armature is due to three causes:

1. The C^2R loss in the armature conductors caused by the useful current.

2. Hysteresis in the iron core.

3. Eddy, or foucault, currents in the core, and sometimes in the conductors.

It is found in average practice that from 1 to $1\frac{1}{4}$ watts for each square inch of external surface is lost in the cores of armatures as they are usually built, due to the effect of hysteresis and eddy currents in the iron discs. When the radiation of the heat thus developed has been taken care of, a margin of one watt per square inch remains for the allowable loss of energy due to the useful current flowing through the copper conductors. In order that the required area for the external surface of any projected armature may be determined, it is necessary to know how many watts due to the C^2R loss are allowable. The loss must be determined from considerations of economy in construction. On the one hand, a reduction of the efficiency and an increase in the size of the armature core are required on account of increasing the C^2R loss, but it results in a saving of copper; on the other hand, an increased amount of copper must be used as R is decreased, with a consequent increase in cost of manufacture. The best results for dynamos designed for ordinary use, probably, can be gained by an examination of the machines of standard makers.

The accompanying table gives the average C^2R losses in armatures of various outputs, as allowed by a number of the best American manufacturers. The losses are given in percentages of the total output, and are based on the *cold resistance* of the armatures. The actual losses at full load will be greater than those tabulated by an amount proportional to the increase of R due to the rise of temperature (= 1 per cent of its value for each $2\frac{1}{2}°$ C. rise of temperature).

Table of C^2R Loss in Armatures according to Average American Practice.

R is taken as Cold Resistance at about 25° C. (75° F.).

Capacity of Dynamo.	Loss in Per Cent.
5 Kilowatt	4.0
10 "	3.4
15 "	2.9
20 "	2.7
25 "	2.5
30 "	2.4
35 "	2.3
50 "	2.2
75 "	2.1
100 "	2.0

R, and therefore the loss, increases 1 per cent of its value for each $2\frac{1}{2}°$ C. rise in temperature.

The table is based upon the assumption of a periphery (circumferential) velocity of 3000′ per minute. A velocity as great as 4000′ per minute is sometimes used, but for mechanical reasons the tendency is towards smaller

velocities. Where dynamos are designed to do special duty requiring extra mechanical stability, or low speed, or both, a velocity as low as 1600′ or 1700′ per minute is sometimes used. Three thousand feet per minute can be considered as a fair average for ordinary conditions. It is evidently desirable to make the velocity as great as mechanical or other limiting conditions will permit, as the output for a given armature varies in direct proportion to its speed. The C^2R loss in an armature varies inversely with the speed, if the output be kept constant. Thus, in the case of a 10-kilowatt (10 K.W.) armature, running at 3000′ periphery velocity, 3.4 per cent loss is good practice. If the same armature be run at 1000′ additional velocity and the output not increased, either the current, and the size of the conductors in a proportional degree, or the number of armature conductors, must be reduced. By either operation the C^2R loss is reduced in proportion to the increase of the speed, and becomes 2.55 per cent. If the armature conductors are not changed, but the output is allowed to vary with the speed, evidently the actual watts in C^2R loss remains constant, and the per cent varies inversely with the speed, as before.

The value of the periphery speed seems to have no marked effect upon the allowable C^2R loss per square inch of armature surface. Thus, in machines without special ventilation, experimented on by Rechnieski, small changes of the periphery speed made no apparent difference in the cooling effect. The increased cooling effect of faster rotation therefore probably compensates, in such machines, for increased eddy current and hysteresis

losses. When special ventilation is arranged, the effect is still less marked. Special ventilation makes a large difference in the heating of armatures. Thus, Rechnieski found the rise of temperature in a ventilated armature to be only about 75 per cent of the rise in a similar but unventilated one.

In order that the current may be safely carried by the armature conductors, experience shows that the density of the current should not be greater than about 2500 amperes per square inch of conductor cross-section. This is equivalent to about 525 cir. mils per ampere : 600 cir. mils is a better and quite usual constant, and 650 cir. mils per ampere is frequently used in designing unventilated armatures of large dynamos or machines for special purposes. In special cases, the current density in armature conductors may be made as great as 4000 amperes per square inch or about 325 cir. mils per ampere, but average conditions demand lower densities.

The loss of energy due to hysteresis has already been shown to vary approximately as $B^{1.6}$, and also as the number of cycles per second. It is thus evident that a limiting value of B in armatures may be reached, which cannot be exceeded without causing excessive heating. This is accentuated by the increase of eddy currents, which are in direct proportion to B, and the heating caused by them is, therefore, proportional to B^2. In some dynamos running at a low speed the value of B in the armature is as great as 15,000 or 16,000 C.G.S. units. With so high a value of B, a careful selection of the quality of the iron is particularly important. Under ordinary circumstances, the value of B is likely to vary

from 6000 to 12,000 lines per square centimeter; the value chosen in any case is determined by such conditions as the quality of iron to be used, diameter of armature, method of manufacture, speed, etc. For the average Siemens armature, 10,000 lines per square centimeter may be considered as a fair value, while the induction usually runs about 12,000 in Gramme armatures. All data necessary to determine the size of an armature for a given duty is now at hand.

The determination of the ratio of length to diameter depends upon the mechanical conditions to be met. It is evident from the formula

$$E=\frac{SN_aV}{10^8\times 60}=\frac{3000\times S\times N_a}{\pi d\times 10^8\times 60}$$

that when a fixed periphery velocity of 3000 feet per minute is determined upon, N_a and d may be changed proportionally, and E will remain constant. If B remains constant, the sectional area of the core is constant, and h (the length) therefore varies inversely with d. Hence, in drum armatures, when d is less than h, if it is decreased while the cross-section is kept constant, the number of conductors will be decreased proportionally and the length of one turn of the winding will be increased in a less proportion. Hence the length of wire will be decreased in some degree, but the revolutions per minute must be changed in inverse ratio with d. This is too great a mechanical sacrifice to allow for the purpose of saving a small percentage in the copper. When d is greater than h, if it is increased, the number of conductors is increased proportionally, and the

length of wire is increased in a somewhat less ratio. Hence the total length of the windings increases more rapidly than the revolutions per minute decrease, and the extra cost for a reduction of speed is excessive. It is thus shown that the most satisfactory ratio of length to diameter for general purposes is about $\frac{h}{d}=1$. Special conditions, however, are often met which make a considerable alteration of this ratio advisable. In some cases h has been made as great as $3d$ in Siemens armatures. For Gramme armatures, the case is more complex, and convenience of construction usually dictates the relative dimensions. The ratios are usually between $\frac{h}{d}=\frac{1}{2}$, $\frac{h}{d}=2$, and $\frac{b}{d}=\frac{1}{5}$, $\frac{b}{d}=\frac{1}{3}$, b being the radial depth of the core. The choice of ratios in any case depends upon the capacity of the machine and the service for which it is designed.

The number of segments or divisions in the commutator requires careful consideration. A large number of divisions makes a commutator which is expensive to build; but if the number be too small, unsatisfactory results, due to sparking and consequent burning, are obtained. The number of divisions must be governed in some degree by the pressure generated. Where an arc is started, as by the spark at the commutator, it becomes destructive in proportion to the pressure between the bars. Hence, for dynamos generating 125 volts or less, the average pressure between adjacent commutator bars should not exceed 6 volts, 5 volts being good practice. In machines generating pressures as high as 1200 volts, too great a number of divisions is required by the limit

of 6 volts between bars, and as much as 25 or 30 volts is sometimes allowed. Since each half of an armature generates the full pressure, a 125 volt armature should have at least 42 coils and an equal number of commutator divisions. Not less than 50 coils would be better.

Commutator bars are usually made of cast or drop-forged copper, and are preferably insulated from each other, and from the armature shaft, by mica.

The following example will show the procedure to be followed in designing an armature.

Example of Armature Calculations.

In the case of a 25 K.W. dynamo, the table shows $C^2R = 2.5$ per cent of the output $= 625$ watts. Hence the armature should have an external surface of 625 square inches. With no special mechanical condition to meet, the best cross-section for a Siemens armature core is square; hence $d = 11.5$ and $A = 132$. The insulating material between the plates and the space occupied by the shaft compose about 20 per cent of the core in the average armature, leaving 80 per cent of iron; hence the effective area of this core is $132 \times .80 = 106$ sq. in. $= 684$ sq. cm. If $B_a = 10{,}000$ C.G.S. lines per square centimeter, N_a is 6,840,000 C.G.S. lines. At a periphery speed of 3000′ per minute, the number of revolutions per minute is $V = 1000$. From the formula

$$S = \frac{60 \times E \times 10^8}{6{,}840{,}000 \times 1000}$$

is derived at once the number of conductors on the periphery of the armature. If 125 volts be desired at

the brushes, $S = 110 -$, or there must be a total of 55 turns on the armature. On account of the loss of pressure in the armature, etc., it is desirable to add a few extra conductors, and for the best forms of winding an even number of turns is desirable. Fifty-six turns may therefore be chosen, and these can be wound on the core as 56 coils of 1 turn each, 28 coils of 2 turns each, etc. Choice must rest upon the conditions likely to give the least commutator sparking and the most compact winding. The armature under consideration should therefore be wound with 56 coils of 1 turn each.

The circumference of the armature core is 36.13 inches, in which space must be wound 112 wires insulated with cotton, and sufficient space be allowed for fibre insulation between the coils. The latter may be from .02 to .05 inch in thickness, and in the first calculation of the diameter of wires, .04 inch is a fair value to use. The cotton insulation on the wires consists of two opposite windings of cotton thread measuring about .016 inch in total thickness on medium sized wires. Fifty-six coils of 1 turn, 1 layer deep, use 112 fibre insulations, or in the circumference $112 \times .04 = 4''.48$. One hundred and twelve wires wrapped with cotton .016 inch thick require for the cotton $112 \times .016 = 1''.79$, making a total of $6''.27$ occupied by insulation, and leaving $29''.86$ for copper, or 112 wires of 267 mils diameter. The nearest wires drawn by American makers are number 2 B. and S. gauge (diameter 258 mils, section 66,400 cir. mils), and number 4 B.W.G. (diameter 238 mils, section 56,600 cir. mils). The dynamo under consideration has an output of 200 amperes, or 100 amperes in

each conductor, which at 600 cir. mils per ampère requires a conductor of 60,000 cir. mils. The number 2 B. and S. wire is therefore of proper cross-section, and it may be used if the thickness of the fibre is properly corrected so as to exactly fill the circumference of the drum. It is to be remembered that the circumference to be filled has a diameter equal to that of the drum with insulation plus the diameter of the insulated wire.

A large wire of circular section is uneconomical of space and hard to wind. Therefore some manufacturers divide the wire into two or more wires wound in parallel, each usually not exceeding in size a number 8 B. and S. gauge wire. In this case, in order to determine the exact size of wire and thickness of fibre to be used, the circumference on which it is to be wound must be determined. The diameter of the armature body is 11.5 inches. The body must be properly insulated, before winding, by a covering of shellacked canvas, rubber tape, oiled paper, and japan, or something similar. This may be estimated to average .025 inch in thickness, in other words; .05 inch is added to the diameter of the body. The diameter of the wire may be approximated as .2 of an inch. Hence the diameter of the circumference on which the wire is wound is $11.50 + .05 + .20 = 11.75$ inches, and the circumference is 36''.91. Of this, 4''.48 is used for fibre, leaving 32''.43 for conductor, or 290 mils per conductor. When this is divided into two wires laid side by side, it gives 145 mils per wire, of which 129 mils may be copper. Number 8 B. and S. gauge wire has a diameter of 128.5 mils, and a cross-section containing 16,509 cir. mils. To make the cross-

section required, demands 4 of the number 8 wires, and they can be wound two wide and two deep, thus $\|{}^{\circ\circ}_{\circ\circ}\|{}^{\circ\circ}_{\circ\circ}\|$. Four number 8 B. and S. wires give a cross-section containing 66,000 cir. mils = 660 cir. mils per ampere.

The external diameter of a finished armature must include the thickness of the binding wires with insulation under them, in addition to the insulated diameter of the core and the wires. The size of the binding wires depends to some extent upon the size of the armature. They are usually made of hard drawn brass or german silver. For a 25 K.W. armature the size of binding wires can be taken as number 18 B. and S. gauge, which has a diameter of 40 mils. Four or five bands from ¾″ to 1″ wide must be put upon the armature. The insulation under the bands is usually made up of mica strips, oiled paper, or fibre, and probably averages in thickness about .020. Thus the 25 K.W. armature of the example will finish to a diameter of, say, $11.500 + .050 + .578 + .120 = 12\frac{1}{4}$ inches.

To determine whether the C^2R_a loss is approximately that assumed, it is necessary to find the length of wire on the armature. The length of a turn equals approximately the perimeter of the armature + 15 per cent allowance for piling on heads, connections to commutator, etc. This gives 53″ per turn, and 56 turns = about 250′. 250′ of number 8 B. and S. wire measures .157 ohms. There are 4 wires in parallel in each turn; hence the total resistance is one fourth of .157 ohms. As the two halves of the armature are in parallel, this is quartered again, and the cold resistance is $\frac{.157}{16} = .010$ ohms; and when the armature is heated to 40° C. above ordinary air temperature, the

resistance becomes .012 ohms. With 200 amperes flowing $C^2R_a = 2.0$ per cent of total output with the hot resistance, which is less than the allowable loss for a 25 K.W. armature given in the table.

The following additional examples will show how far the constants are likely to vary in actual machines. The first example is a Phœnix dynamo designed by Mr. W. B. Esson, who is a believer in excessively high periphery velocities. This accounts for the high velocity (3950 feet per minute). The data for this machine are taken from Thompson's *Dynamo-Electric Machinery.* The other examples are standard dynamos built by different American makers of high standing.

1. Phœnix (English) dynamo: capacity, $9\frac{1}{2}$ K.W.; normal pressure, 105 volts; normal maximum current, 90 amperes; Gramme armature; external diameter of armature core, $d = 10\frac{5}{8}'' = 10''.625$; internal diameter of core, $d' = 8''.0$; radial depth of core, $b = 1\frac{5}{16}'' = 1''.312$; length of core, $h = 9''.0$; net length of iron in core, $8''.312$; effective area of core, $A = 22$ sq. in. $= 142.26$ sq. cm.; revolutions per minute, $V = 1420$, giving a periphery velocity of 3950′ per minute; external area of core $= 377$ sq. in.; cold $C^2R_a = 324$ watts $= 3.4$ per cent, giving $1\frac{1}{6}$ square inches per watt dissipated. Number of commutator divisions, $2a = 36$; each coil consists of 5 turns, making number of conductors on surface, $S = 180$; windings made with square wire 0.180 square inches, making an area equivalent to 28,640 cir. mils; cir. mils per ampere $= 637$; $N_a = \dfrac{10^8 \times 60 \times 105}{180 \times 1420} = 2{,}645{,}000$ C.G.S. lines. $B_a = \dfrac{N_a}{142} = 17{,}400$ C.G.S. lines per square centimeter. The large value of B_a in this dynamo is to be noted.

It is usual to make B_a slightly greater in Gramme armatures than in Siemens armatures (see page 111), but so large a value is likely to cause excessive heating, unless special care is taken in the selection of the iron for the core and in the arrangements for ventilation.

2. American dynamo: capacity, 22.5 K.W.; normal pressure, 125 volts; normal maximum current, 180 amperes; Siemens armature; diameter of armature core, $d = 10\frac{7}{16}'' = 10''.438$; length of core, $h = 11.''5$; effective area of core, $A = 96.4$ sq. in. $= 621$ sq. cm.; revolutions per minute, $V = 1300$, giving a periphery velocity of 3550 feet per minute; external area of core $= 508$ sq. in; cold $C^2R_a = 259$ watts $= 1.15$ per cent, giving $2\frac{1}{12}$ square inches per watt dissipated; number of armature coils = number of commutator divisions, $2a = 40$; each coil consists of two turns, making number of conductors on surface, $S = 160$; windings made with two number 6 B. and S. wires (.162—.178) in parallel, giving an area of 52,500 cir. mils; cir. mils per ampere $= 583$; $N_a = \frac{10^8 \times 60 \times 125}{160 \times 1300} = 3{,}600{,}000$ C.G.S. lines; $B_a = \frac{N_a}{621} = 5800$ C.G.S. lines per square centimeter.

3. American dynamo: capacity, 10 K.W.; normal pressure, 125 volts; normal maximum current, 80 amperes; Siemens armature; diameter of armature core, $d = 6''.25$; length of core, $h = 12''.0$; effective area of core, $A = 60$ sq. in. $= 387$ sq. cm.; revolutions per minute, $V = 1600$, giving a periphery velocity of 2620′ per minute; external area of core, 296 square inches; cold $C^2R_a = 384$ watts $= 3.8$ per cent, giving $\frac{3}{4}$ square inch

per watt dissipated; number of armature coils = number of commutator divisions, $2a=50$; each coil consists of 2 turns, making the number of conductors on the surface, $S=200$; windings made with number 9 B. and S. wire (.148—.164), giving an area of 21,900 cir. mils; cir. mils per ampere $=548$; $N_a=\frac{10^8\times 60\times 125}{200\times 1600}=2,345,000$ C.G.S. lines; $B_a=\frac{N_a}{296}=9000$ C.G.S. lines per square centimeter.

A somewhat different method of predetermining the constants of an armature is given by Monnier. Starting with the formula $E=\frac{SNV}{10^8\times 60}$, the resistance of the armature is assumed. If the length and diameter of the armature core (using a Siemens armature, for example) be represented by h and d, then the lengths of a turn can be taken as $2h+3d$, and the following equation is derived:

$$R=\tfrac{1}{8}\frac{S(2h+3d)\theta}{\Delta},$$

where Δ is the cross-section of the wire in cir. mils, and θ the resistance of a milfoot. If δ be the diameter of the wire of which the winding is composed, and $m\delta$ its insulated diameter, then

$$Sm\delta=\pi d.$$

Since the C^2R loss in the armature is transformed into heat, the following conditions exist. In any length of the armature conductor, as λ, the resistance is $\frac{\lambda\theta}{\Delta}$, and the heating, when the armature current is C, is

$.24 \times \frac{C^2}{4} \times \frac{\lambda\theta}{\Delta}$ for each layer of wire. If the armature be wound with q layers, the total heat developed in a portion of the windings of length λ and of width $m\delta$, is $\frac{.24 q C^2 \lambda\theta}{4\Delta}$.

If t be the maximum safe rise of temperature above the surrounding air, and j the amount of heat dissipated per unit surface and per degree difference of temperature, the following relation exists:

$$\frac{.24 q C^2 \lambda\theta}{4\Delta} = \lambda m\delta j t.$$

Letting γ represent the density of current in the armature conductors in cir. mils per ampere, there results $C = \frac{2\Delta}{\gamma}$, whence $\frac{q\Delta}{\gamma m\delta} = \frac{jt}{.24\theta} = K =$ a constant.

If for q there be substituted the depth of winding in mils, p, divided by $m\delta \left(\text{i.e. } q = \frac{p}{m\delta}\right)$,

there results $$\frac{p\Delta}{\gamma\delta^2 m^2} = \frac{p}{\gamma m^2} = K.$$

The equation $$EC = \frac{SNVC}{10^8 \times 60}$$

then becomes $$EC = (2uBm\delta K)hd,$$

where u is the periphery velocity, and

$$C^2 R_a = 2\frac{\pi m\delta\theta}{\gamma^2}(2h + 3d)\,d.$$

These equations may be written

$$hd = O,$$

$$(2h + 3d)\,d = P,$$

in which O and P are known quantities when the values of E, C, R_a, u, B, K, m, γ and δ are given by the conditions, or have been assigned such values as experience warrants.

Solving gives $$h=\sqrt{\frac{P-3O}{2}},$$

and $$d=\sqrt{\frac{2O^2}{P-3O}}.$$

A similar solution may be made for the Gramme ring, resulting thus:

$$hb=O,$$

$$(h+b)d=P,$$

requiring a solution by approximation to find the values of the unknown quantities.

This is an unwieldy method of determining the relations of b, h, and d, and would usually give results of little value, since they must almost always be adjusted to conform to such conditions as may arise in the course of a design.

CHAPTER V.

THE MAGNETIC CIRCUIT OF THE DYNAMO.

WITH the calculations for the armature of a dynamo completed, the calculation and design of the field magnets can be at once entered upon. The object of the field magnets is to afford a magnetic circuit in which a magnetic pressure may be placed for the development of the lines of force which are required to pass through the armature. The total magnetic circuit, or path, of the lines of force may be considered as made up of three parts: the field magnets or *frame*, the air space or *gap*, the armature *core*. The reluctance of the air gap is always a large percentage of the total reluctance of the circuit. It sometimes amounts to more than 90 per cent of the total reluctance. Since the specific reluctance of air is unity, there must be considerable leakage of magnetic lines of force directly across from pole-piece to pole-piece, and therefore around the air gap and armature (see Fig. 54 and compare page 10).

The leakage paths are in parallel with the useful path for the lines through the armature, and the number of lines of force in each path is inversely as the reluctances and directly as the magnetic pressure (compare page 8).

The laws of parallel circuits can therefore be applied to the paths for lines of force exactly as they are applied

to electric circuits. The conditions to be met in the two cases are usually quite different, however. Thus, electric circuits are usually in the form of wires, the dimensions of which are readily measured, and the currents are confined to them; while leakage circuits for magnetic lines of force usually terminate in two surfaces of more or less indefinite form, and the exact areas of the paths are quite indeterminate. The great area of the surfaces in which the leakage paths terminate reduces the total leakage reluctance to a magnitude directly comparable to that of the useful path through the air gap and armature. To know the number of leakage lines is therefore a matter of moment in determining the reluctance of the frame, and hence in calculating the field windings. The forms of leakage paths, as ordinarily met with in dynamos, may be reduced by approximation to a comparatively few simple ones, and the total leakage reluctance can be calculated with some degree of approximation.

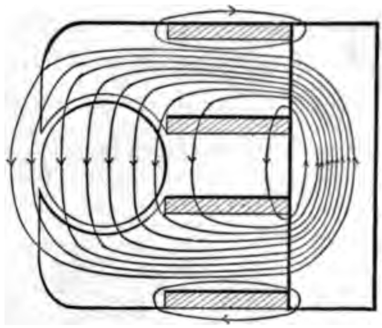


Fig. 54.

Leakage Reluctance (P_l).*

The average lengths of the paths of the leakage lines are readily determined in most cases in practice, with a

* See Vol. 15, *Jour. Soc. Tel. Eng.*, Forbes's discussion on Kapp's paper; Picou, *Machines Dynamo-Electrique*, p. 135; Kapp, *Electric Transmission of Energy*.

fair degree of accuracy, consequently it is necessary to make limiting assumptions regarding only the areas of the paths.

Conceive two surfaces facing each other in air, and a mean distance apart l, and divided into elementary areas dA. The magnetic conductivity of each tube between opposite elements is $\frac{dA}{l}$, and the magnetic conductivity between the surfaces is $\Sigma \frac{dA}{l}$, or $\frac{1}{P} = \int\int \frac{dA}{l}$.

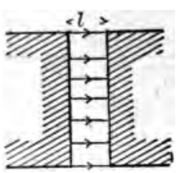


Fig. 55.

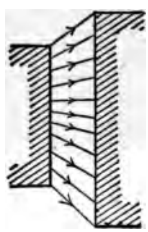


Fig. 56.

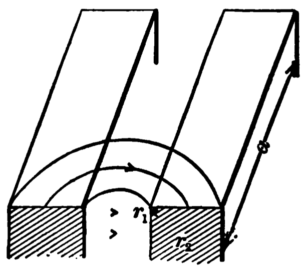


Fig. 57.

If the surfaces are equal planes opposite to each other (Fig. 55), this evidently becomes

$$\frac{1}{P} = \frac{A}{l}, \text{ or } P = \frac{l}{A}.$$

When the surfaces are unequal planes opposite each other (Fig. 56), it is evident that

$$\frac{1}{P} = \frac{A_1 + A_2}{2\,l} \text{ or } P = \frac{2\,l}{A_1 + A_2}.$$

When the surfaces are in the same plane and close together, as shown in Fig. 57, the lines of force may be assumed to be confined between the cylindrical surfaces described with radii r_1 and r_2.

From the figure, $dA = adr$, and $l = \pi r$.

$$\therefore \frac{1}{P} = \frac{a}{\pi}\int_{r_1}^{r_2} \frac{dr}{r} = \frac{a}{.434\pi}\log_{10}\frac{r_2}{r_1} = .734a\log_{10}\frac{r_2}{r_1},$$

or $$P = \frac{1.363}{a\log_{10}\frac{r_2}{r_1}}.$$

When the surfaces in the last case are a considerable distance apart, it is necessary to change the assumed boundaries of the path of the lines of force. One may be taken to be a plane of width b, placed between the edges of the surfaces, and the other as made up of two quarter-cylinders joined by a plane of width b. If r be the width of the surfaces, the quarter-cylinders will have a radius of r.

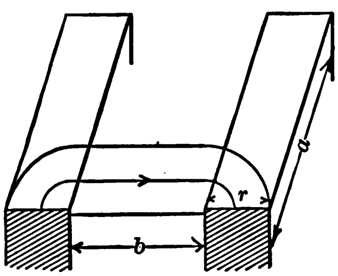


Fig. 58.

From Fig. 58, $dA = adr$, and $l = b + \pi r$. But

$$dr = \frac{1}{\pi}d(b + \pi r).$$

$$\therefore \frac{1}{P} = \frac{a}{\pi}\int_{b}^{b+\pi r} \frac{d(b+\pi r)}{b+\pi r} = \frac{a}{.434\pi}\log_{10}\frac{b+\pi r}{b}$$

$$= .734a\log_{10}\frac{b+\pi r}{b} \text{ or } P = \frac{1.363}{a\log_{10}\frac{b+\pi r}{b}}.$$

With these formulæ for the value of P under various conditions, and that giving the value of P between two cylinders (page 131), nearly all examples arising in

practice can be approximately solved. The total leakage resistance is frequently formed of several paths in parallel, P for each path being separately calculable. An example is shown in Fig. 59, which is a combination of case 2 and case 3.

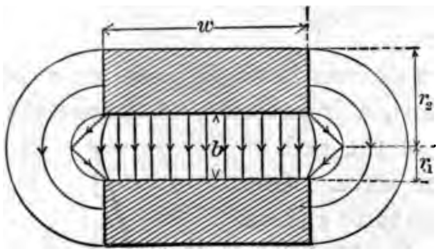


Fig. 59.

The figure shows a cross-section through the legs of a magnet, the dimensions being, height or length a; width w; thickness $r_2 - r_1$; and distance apart $2r_1 = b$.

The average width of the path between the opposite surfaces is

$$\frac{w + (w + 2r_1)}{2} = w + r_1. \therefore A = (w + r_1)a \text{ and } \frac{1}{P_d} = \frac{(w + r_1)a}{b},$$

where P_d is the resistance of the path between the opposite surfaces.

For the path on each side lying between the semicircles

$$\frac{1}{P_s} = .734 a \log \frac{r_2}{r_1}.$$

For the total magnetic conductivity of the leakage path we take the sum of the several conductivities, or

$$\frac{1}{P_l} = \frac{1}{P_d} + \frac{1}{P_s} + \frac{1}{P_s} = \frac{1}{P_d} + \frac{2}{P_s} = X, \text{ and } \therefore P_l = \frac{1}{X}.$$

If the legs had been a considerable distance apart, this example would have become a combination of case 1 and case 4.

To determine the reluctance between two cylinders of radius r, which are parallel and at a distance apart, centre to centre, b, it is most convenient to first find the reluctance of the path between two infinitesimal elements of length. The total reluctance is then readily found by summing up for the whole length of the cylinders. Figure 60 shows the cross-section of two cylinders A and B with centres at c and c'.

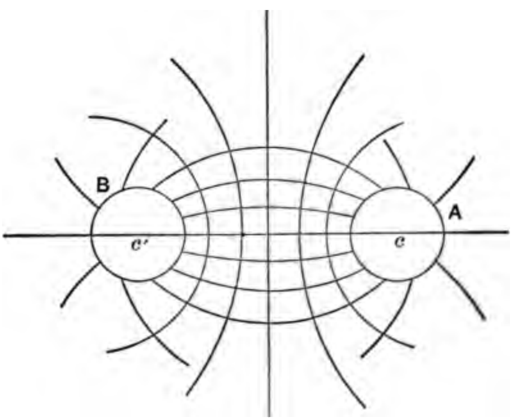


Fig. 60.

Assuming the cylinder A to be at a magnetic potential M_A, and the cylinder B to be at a magnetic potential M_B, then the lines in the figure joining the cylinders show the form of the magnetic lines of force and the other lines show the form of the equi-potential surfaces in cross-section. The cross-section and length of the

average path of the lines of force cannot be directly approximated, as in the simpler problems already passed over; it is therefore necessary to find some expression which represents the magnetic potential M at any point outside of the cylinders, and then by proper substitutions and integrations to solve for P.

At a point of magnetic potential M, the rate of change of potential, or the magnetic force, measured along the direction of the axis of x, is evidently $\frac{dM}{dx}$. At a little distance from the first point measured parallel to the axis of x, the magnetic force is $\frac{dM'}{dx}$. If the distance between the first and second points be infinitesimal, there results $\frac{dM}{dx} - \frac{dM'}{dx} = \frac{d^2M}{dx}$. The number of lines of force passing through a plane area of infinitesimal dimensions, $dydz$, and perpendicular to the axis of x at the first point is evidently $\frac{dM}{dx}dydz$, and the number passing through an equal area at the second point is $\frac{dM'}{dx}dy\,dz$. The difference in the number of lines of force passing through the two areas is therefore

$$\left(\frac{dM-dM'}{dx}\right)dy\,dz = \frac{d^2M}{dx^2}\,dx\,dy\,dz.$$

If this reasoning be applied to an infinitesimal cube with its edges parallel to rectangular axes, along which the total magnetic force is resolved, there is shown to be a difference in the number of lines of force passing through opposite faces of the cube as follows:

$\frac{d^2M}{dx^2}\,dx\,dy\,dz$ for the faces parallel to the Y and Z axes.

$\frac{d^2M}{dy^2}\,dx\,dy\,dz$ for the faces parallel to the X and Z axes.

$\frac{d^2M}{dz^2}\,dx\,dy\,dz$ for the faces parallel to the X and Y axes.

Unless the cube be a magnet, an equal number of lines of force must enter and leave it, hence

$$\frac{d^2M}{dx^2}\,dx\,dy\,dz + \frac{d^2M}{dy^2}\,dx\,dy\,dz + \frac{d^2M}{dz^2}\,dx\,dy\,dz = 0,$$

or

$$\frac{d^2M}{dx^2} + \frac{d^2M}{dy^2} + \frac{d^2M}{dz^2} = 0.^{*}$$

If the magnetic field in which the cube is located be uniform, d^2M is 0, and the number of lines of force passing through opposite faces must be equal. If the cube be a magnet of pole strength "m," $4\pi m$ lines of force must emanate from it (compare page 2), hence there is a difference of $4\pi m$ lines of force between the number entering and the number leaving the cube, or

$$\frac{d^2M}{dx^2} + \frac{d^2M}{dy^2} + \frac{d^2M}{dz^2} = 4\pi m.^{\dagger}$$

In the solution at hand, each line of force may be assumed to be a curve lying entirely in a plane which is perpendicular to the axes of the cylinders. If the axis of z be taken parallel to the axes of the cylinders $\frac{d^2M}{dz^2}\,dx\,dy\,dz$ is equal to zero, for there is no change of potential in that direction, and there results

$$\frac{d^2M}{dx^2} + \frac{d^2M}{dy^2} = 0.$$

---

* Often called Laplace's formula. † Often called Poisson's formula.

Any expression for the potential M that will satisfy this equation will serve in the desired solution for P.

In Fig. 61, if two points be taken within the circles which represent the cross-section of the cylinders, and

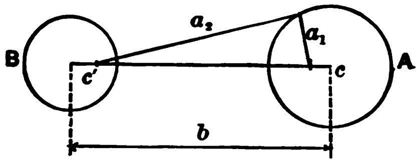


Fig. 61.

on the line joining their centres, so that the ratio of the distances of these points from the circumference of either circle, $\frac{a_1}{a_2}$, is a constant for all points on the circle taken, then the expression $M = -A \log_e \frac{a_1}{a_2} + C$ fulfils the conditions. A and C are constants fixed by the conditions of the problem. The number of lines of force passing through an element of the surface of either cylinder is evidently $dN = \mu \frac{dM}{dr} ds$. The total number of lines of force passing between the cylinders is therefore $N = \mu \int \frac{dM}{dr} ds$, the integration being extended around the boundary of a cylinder.

From $$M = -A \log \frac{a_1}{a_2} + C,$$

$$dM = -A \left(\frac{da_1}{a_1} - \frac{da_2}{a_2} \right),$$

and $$\frac{dM}{dr} = -A \left[\frac{\frac{da_1}{dr}}{a_1} - \frac{\frac{da_2}{dr}}{a_2} \right];$$

hence $$N = -A\mu\left[\int \frac{\frac{da_1}{dr}}{a_1} ds - \int \frac{\frac{da_2}{dr}}{a_2} ds\right].$$

ds may be written $rdld\theta$, where dl is the thickness of the layer under consideration. The integral of the first term is therefore evidently dl times the summation of the angles subtended by the infinitesimal elements of a circumference at a point within it, and is equal to $2\pi dl$. The integral of the second term is, in the same manner, dl times the summation of the angles subtended by a circumference at a point without, and is equal to zero.

Hence $$N = -2\pi A\mu dl = \frac{M_A - M_B}{P},$$

and therefore $$P = -\frac{M_A - M_B}{2\pi A\mu dl} = -\frac{A\log_e\frac{a_1}{a_2} - A\log_e\frac{a_2}{a_1}}{2\pi A\mu dl}$$

$$= \frac{1}{\pi\mu dl}\log_e\frac{a_1}{a_2}.$$

For any other length of cylinder, a summation of dl may be taken to the length desired. For leakage paths in air $\mu = 1$ and

$$P = \frac{1}{\pi l}\log_e\frac{a_1}{a_2} = \frac{\cdot 737\log_{10}\frac{a_1}{a_2}}{l}.$$

Before this formula is available for use, the value of $\frac{a_1}{a_2}$ must be given in terms of the diameter of the cylinders, and their distance apart (centre to centre). This value is

$$\frac{a_1}{a_2} = \frac{d}{b - \sqrt{b^2 - d^2}},$$

where d is the diameter and b the distance apart (centre to centre).

The term $\frac{d}{b-\sqrt{b^2-d^2}}$ does not lend itself to easy computations, but its numerical value is constant for all diameters of the cylinders, as long as the ratio $\frac{b}{d}$ is kept constant. Hence the reluctance between any two parallel cylinders can be readily found by the assistance of a table giving the reluctances between unit lengths of two cylinders for various ratios of $\frac{b}{d}$. A table quite similar to the accompanying one was first given by S. P. Thompson in his lectures on the electro-magnet.

Table showing the magnetic reluctance in C.G.S. units between unit lengths of two equal parallel cylinders surrounded by air, and having various values of the ratio $\frac{b}{d}$.

$\frac{b}{d}$	P PER CM.	$\frac{b}{d}$	P PER CM.	$\frac{b}{d}$	P PER CM.
1.25	.19	4.	.655	7.5	.86
1.50	.30	4.5	.67	8.	.88
1.75	.337	5.	.73	8.5	.90
2.	.42	5.5	.76	9.	.92
2.5	.50	6.	.79	9.5	.94
3.	.556	6.5	.815	10.	.96
3.5	.61	7.	.84		

To determine the reluctance between any two cylinders, find the value of $\frac{b}{d}$ from their dimensions. Take from the table the appropriate value of the reluctance per centimeter, and divide by the length of the cylinders in centimeters.

The application of the leakage formulas may be seen from what follows. Fig. 62 *a* represents the outline of a dynamo frame, the windings of which are on the keeper. In a well-designed and efficient dynamo the reluctance of the portion of the limbs between the keeper and the pole-pieces is usually a quite small fraction of the total reluctance in the magnetic circuit. Hence the total magnetic pressure ($M = 1.25\,\overline{nc}$) can, with sufficient accuracy, be looked upon as acting directly between the

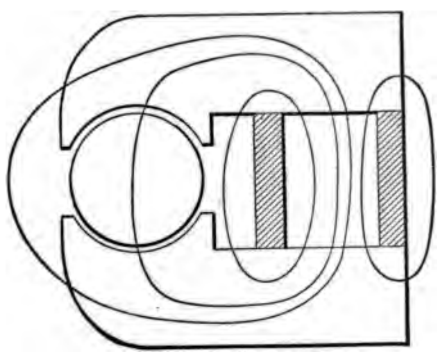


Fig. 62 a.

leakage surfaces. It can also be approximately considered as acting directly between the pole-pieces to generate the useful lines through the armature. With the limitations in accuracy due to these approximations, the following relations are now established. The total leakage lines are, approximately, $N_l = MX_l = \frac{M}{P_l}$, where X_l and P_l represent respectively the magnetic conductivity and reluctance of all the leakage paths in parallel. The useful lines are also approximately $N_a = \frac{M}{P_{a+a}}$, where P_{a+a}

is the reluctance through air gap and armature. The total number of lines in the frame is therefore approximately $N_t = N_a + N_l$, and the ratio between the total lines passing through the frame and the number passing through the armature is $\frac{X_{a+a} + X_l}{X_{a+a}} = \frac{P_{a+a} + P_l}{P_l}$. This ratio, the importance of which was first pointed out by Dr. Hopkinson, is often represented by the letter v, and is called the **Leakage Coefficient**; $v = \frac{N_t}{N_a}$.

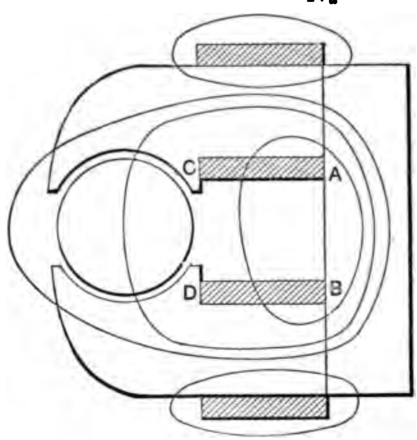


Fig. 62 *b*.

In dynamos where the windings are divided so that one-half the magnetic pressure is developed by a coil on each limb, the number of leakage lines cannot be regarded as directly proportional to X_l. This is evident from Fig. 62 *b*, which shows that only one-half the total magnetic pressure tends to send leakage lines across the space between the limbs; because the difference of magnetic pressure between *A* and *B* can be taken as

approximately zero, and that between C and D as approximately equal to M; hence, the average difference of magnetic pressure between the limbs can be taken as equal to $\frac{1}{2}M$. The necessary correction may be made in calculating X_l, giving a value that we will call X'_l. This can be directly combined with X_{a+a} to find v as before.

If the reluctance of the magnet frame is not comparatively small, as assumed, the magnetic pressure causing leakage through the various paths is sensibly less than M, and must be different for the different paths. The difference of magnetic pressure between the pole-pieces is also sensibly less than M. Hence corrections must be applied to both X_{a+a} and X_l. Errors in estimating the magnetic pressures at various points are evidently eliminated to a large extent from the value of v, for they are likely to affect both numerator and denominator of the fraction $\frac{X_{a+a}+X_l}{X_{a+a}}$ in about the same proportion. Moreover, the calculated value of v cannot be expected, and is not required, to be brought to an accuracy closer than 15 to 20 per cent of its true value in the finished machine.

The surfaces terminating the leakage paths of actual dynamos are frequently irregularly curved and stand in various planes. It therefore is necessary, in such cases, to assume an average surface to be used in calculating.

The windings of the field magnets of dynamos are usually classified according to their arrangement in circuit. The principal divisions are two: *separately excited*

and *self-excited*, so called, respectively, when the magnetizing current is supplied from an external source or from the armature of the machine under consideration. Self-excited dynamos are again divided into *series*

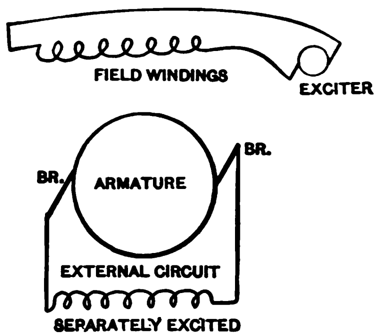


Fig. 63.

wound, *shunt wound*; and *compound wound*, depending upon whether: first, the whole current is led through a comparatively few turns around the field magnets;

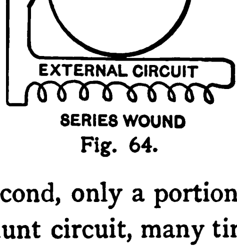


Fig. 64.

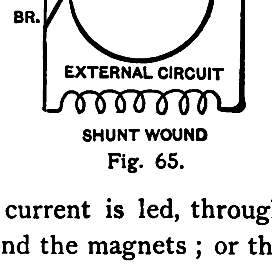


Fig. 65.

second, only a portion of the current is led, through a shunt circuit, many times around the magnets ; or third,

a combination of the first two. The third division, or compound winding, can be further subdivided according to the arrangement of the shunt winding. If this is connected around the series coil, as in Fig. 66, the compound winding is said to be *long shunt*, and it is *short shunt*, when the winding is connected directly from brush to brush, as in Fig. 67. The purposes to which the different forms of winding are usually applied will be discussed later (Chaps. VII. and VIII.).

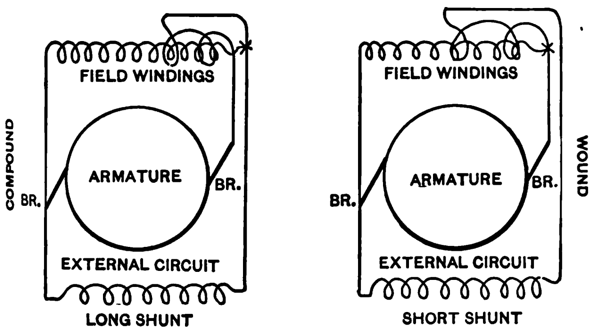


Fig. 66. Fig. 67.

The forms of two-pole field magnets are numberless, but in the standard machines they may be reduced to two principal classes: first, *single horseshoe* type; second, *double horseshoe* type. Each of these may be subdivided into three typical forms, depending upon the position of the *windings* or *coils*: first, coils on limbs; second, coil on yoke; third, coils on both. The third division is not as often used as the others.

In designing the windings of field magnets, a good foundation from which to start is the limit of C^2R loss

and of heating, as in the case of an armature; hence, a table of C^2R losses allowable in good practice follows. This is made up in the same manner as the table of armature C^2R losses given on page 108.

Table of C^2R Loss in Field Windings of Shunt and Series Dynamos.

R being taken as Cold Resistance at about 25° C. (75° F.). Per cent of Full Load according to Average American Practice.

CAPACITY OF DYNAMO.	LOSS IN PER CENT.
5 Kilowatts	4.5
10 "	3.6
15 "	3.1
20 "	2.8
25 "	2.5
30 "	2.3
35 "	2.1
40 "	1.9
50 "	1.7
60 "	1.5
75 "	1.3
100 "	1.1

R, and therefore the loss, increases 1 per cent of its value for each 2½° C. rise in temperature.

The armature for a given machine having been fully determined, the area of the iron in the field magnet is fixed by the fraction $\frac{N_a v}{B_f}$. For wrought iron, in the best practice, B_f varies from 15,000 to 18,000 lines per square centimeter, depending upon the quality of the iron and the conditions for which the dynamo is designed. For average commercial machines, 16,000 lines per

square centimeter can be taken as a satisfactory value. For cast iron B_f varies in the best practice from 6000 to 8000 lines per square centimeter, and sometimes reaches 10,000 lines per square centimeter. For average commercial machines, 8000 lines per square centimeter can be taken as a satisfactory value. It is evident that the value of B in the field magnets is determined by considerations of economy of manufacture. Thus, to decrease B, it is necessary to increase the cross-section, and therefore the weight of the iron, of the fields. At the same time, the reluctance is decreased, which decreases $\overline{nc}$, though the length of each of the turns in the windings is made greater. Hence, to decrease B, requires an increase in the amount of iron but effects a saving in copper. On the other hand, to increase B effects a saving in iron but causes an increased use of copper. A balance, which depends upon the relative cost of iron and copper in the finished machine, is sought.

The value of v evidently depends upon the form of the frame and the value of $P_{a+a'}$. As the reluctance of the air space is the major part of $P_{a+a'}$, v evidently depends upon the dimensions of the air space and of the leakage paths. Hence, if v be determined for one machine of a given type, it will have practically the same value for all other machines of the same type whatever their dimensions, provided the linear dimensions of the individual machines are always in the same proportion. In plain single horseshoe fields, v usually lies between 1.25 to 1.4 for the most efficient machines. In inverted single horseshoe (Edison) and plain double

horseshoe fields, v usually lies between 1.5 and 1.75. In making a first estimate of the size of cores, 1.3 and 1.6 are satisfactory values of v to use for the respective types of fields. Since the reluctance of the air space P_a is a large portion of P_{a+a}, it is evident that v will be a practical constant for a particular dynamo, over the whole range of saturation in practical working. The following table gives leakage tests made on two machines at various magnetizations.

10 K.W. Machine.		20 K.W. Machine.	
Magnetizing Power.	v.	Magnetizing Power.	v.
1	1.59	1	1.26
2	1.59	2	1.31
3	1.57	3	1.33
4	1.56	4	1.36
4⅓	1.57	5	1.33

The range of magnetizing power used in the tests from which these tables are drawn, starts considerably below the normal magnetization and passes considerably above it.

With a fixed sectional area of core and a fixed number of turns in the winding, a cylindrical core will evidently require the shortest length of wire in the windings, and therefore the least weight of copper. Where economy in manufacture is important, a cylindrical core is advantageous, as it lends itself readily to machine-shop practice. For these reasons, single horseshoe magnets usually have cylindrical cores. Exam-

ples: Edison dynamo, Thomson-Houston motor type dynamo, National dynamo. When the cores and pole pieces are forged from one piece, it is more economical to make the cores square with rounded corners. Example: Edison-Hopkinson dynamo (English). Consequent pole machines with the windings on vertical cores usually have cylindrical cores. Examples: Sprague motor, Rae dynamo, Manchester dynamo (English). When the winding of consequent pole frames is on horizontal cores, the cores are usually elliptical in section for mechanical reasons. Examples: Weston dynamo, Westinghouse horizontal type dynamo.

Besides the general cases given above, there are numerous unclassed forms of dynamo fields with either cylindrical or elliptical cores. Examples of cylindrical cores: Gramme dynamos of certain types, Gülcher dynamo, Deprez dynamo, United States motor, etc. Examples of elliptical cores: Mather dynamo, Brush dynamo, various street railway motors, etc. In other dynamos the magnet cores are made from slabs or bars cut from merchant stock. Examples: some Siemens and Crompton dynamos. In the best dynamos, the form of core and frame is dictated by the required mechanical or electrical duty, economy of manufacture being considered at the same time. In some instances the form has been due to a whim of the designer, but whimsical dynamos cannot succeed in commercial work, on account of economical considerations. Dynamos must be designed with due consideration for economy of manufacture and for the conditions of their service. Dynamo designing is as much a science as steam engine designing

and it is as fully dependent on sound theoretical principles. As in the case of the engine, theory must be guided by judgment and experience.

For the determination of field windings, we have

1. $A = \frac{N_a v}{B_f}$.

Experience shows that .35 to .40 watts radiated per square inch of outer surface raises the temperature of the field coils about 30° to 40° C. (70° F.), which is as high as the temperature should go in good practice. Thirty-five to forty hundredths of a watt per square inch of outer surface of the coils is about equal to one-half of a watt per square inch of core surface on which the wire is wound. Hence we have

2. Length of core is such that one-half a watt (at loss given in table, page 138) will be radiated for each square inch of surface on which wire is wound.

3. $R_f = \frac{\text{output} \times \text{per cent loss (table)}}{C^2}$

$= \frac{E}{C} \times$ per cent loss. For series dynamos.

$R_f = \frac{E}{C_f}$, and $C_f = C \times$ per cent loss, making

$R_f = \frac{E}{C \times \text{per cent loss}}$. For shunt dynamos.

The number of ampere turns is determined from the values of N and P, as already demonstrated, and the number of turns of wire is directly deduced. The mean

length of a turn being determined, the diameter of the conductor can be deduced.

The depth of wire on a core can be taken approximately as $\frac{1}{4}r$, r being the mean radius of the core. On some machines this becomes as great as $\frac{1}{3}r$, and on dynamos of capacity less than 5 K.W. it is still larger.

If the depth of winding is assumed to be $\frac{1}{4}r$, the mean length of a turn is $l=\frac{9}{4}\pi r=\frac{9}{8}\pi d$. Total length of wire $L=tl$, where t=number of turns. Since $R=\frac{L\theta}{\overline{cm}}$, where θ is the specific electric resistance and $\overline{cm}$ is the sectional area of the conductor, we have $\overline{cm}=\frac{L\theta}{R}$. As $\overline{cm}$ is desired in circular mils, while L is in feet, and R in ohms, it is necessary for θ to be the resistance of a wire one foot long and one circular mil in area, *i.e.* the resistance of one "mil foot," $=10.6$ ohms at ordinary temperatures, hence $\overline{cm}=\frac{10.6\,L}{R}$. From wire tables, the gauge number and diameter of the conductor are at once determined when $\overline{cm}$ is known.

As a check upon the magnetic determinations, it is best to calculate the value of B in the air space. This should usually be within the limits of 3000 to 6000 C.G.S. lines per square centimeter.

Example in Determination of Field Winding.

Let it be required to wind fields for the armature of the previous example, the fields to be of the Edison type, and $v=1.6$.

$$N_f=N_a\times 1.6=6{,}840{,}000\times 1.6=10{,}944{,}000.$$

Use 10,950,000.

$B_f = 16{,}000$. $\therefore A = 685$ sq. cm., and $D_f = 29.5$ cm.

29.5 cm. $= 11''.625$ ($11\frac{5}{8}''$).

The cores of the Edison type are cylindrical, and in this machine should be $11\frac{5}{8}''$ diameter of wrought iron. The table shows that a 25 K.W. machine may have a loss of 2.5 per cent in the field windings $= 625$ watts. Each core therefore should have 625 square inches external surface, making its length $b_f = \frac{625}{\pi D_f} = 17\frac{1}{8}''$. The ends of the winding should be protected by a fibre ring from $\frac{3}{16}''$ to $\frac{1}{2}''$ thick. $\frac{1}{4}''$ is a good average thickness; and as there is a ring at each end, $\frac{1}{2}''$ in length is occupied by fibre, leaving $16\frac{5}{8}''$ ($16''.6$) for winding space. The mean length of a turn $l = \frac{9}{8}\pi \times 11\frac{5}{8}'' = 41''.1 = 3'.42$.

We will assume that the frame is laid out on the drawing board and the value of v checked by application of the leakage formulæ. The ampere turns required on the field must next be calculated by the methods given on pages 7, 43, etc. Thus the magnetic pressure required to force N_a lines of force through the reluctance of the armature core and air space is $M_a = N_a P_{a+a}$. The magnetic pressure required to force N_f lines through the frame is $M_f = N_f P_f$. Hence the total magnetic pressure is $M_a + M_f = N_a P_{a+a} + N_f P_f = 1.25\ \overline{nc}$, and hence $\overline{nc} = \frac{N_a P_{a+a} + N_f P_f}{1.25}$. The values of the various reluctances are calculated from the estimated cross-sections and average lengths of the paths of the lines of force. The lengths of the paths are most readily found

by direct measurement from the drawing board. The average length of the lines through the armature is greater than the diameter of a Siemens armature, as the lines are required to spread on account of the shaft hole in the discs. It is therefore well to take the length as equal to the line e in Fig. 68. This is likely to be larger than the actual average length, but it errs on the safe side, and P_a is always small in comparison with $P_{a'}$, so that errors in it do not greatly influence the result. The length of the air space is evidently the difference between the diameter of the armature core and the bore of the pole pieces. Its cross-section is found by taking the product of the length of the armature core times the length of the arc of the pole pieces. To this should be added about 10 per cent to allow for the "fringing" of the lines of force at the corners of the pole pieces. The reluctance of the frame is made up of the reluctances of the pole pieces, cores, and yoke. These are not often of the same cross-section or quality of iron, and hence their reluctances must be calculated separately. Thus $P_f = P_p + P_c + P_k$. The lengths of the average paths in the parts of the frame are readily taken from the drawing board, and the cross-sections are found from the known values of N_f and B_f. If the quality of iron differs in different parts of the frame, B_f must vary accordingly. Evidently

$$P_p = \frac{l_p}{\mu_p A_p}, \quad P_c = \frac{l_c}{\mu_c A_c}, \quad P_k = \frac{l_k}{\mu_k A_k},$$

and

$$P_f = \frac{l_p}{\mu_p A_p} + \frac{l_c}{\mu_c A_c} + \frac{l_k}{\mu_k A_k}.$$

In the example under consideration, we will assume the determination of M to be completed, giving $\overline{nc} = 17{,}500$. The output in current of the dynamo being 200 amperes, the field current is 5 amperes ($= 2.5$ per cent of 200) for a shunt dynamo, and therefore a total of 3500 turns is required, or 1750 turns is required per core. Therefore

$$L = 3.42 \times 3500 = 11{,}970 \text{ feet},$$

$$R_f = \frac{125}{5} = 25 \text{ ohms},$$

$$\overline{cm} = \frac{11{,}970 \times 10.6}{25} = 5075 \text{ cir. mils},$$

which is very nearly equal to the area of a number 13 B. and S. wire. The "covered diameter" of number 13 B. and S. wire is 80 mils when d.c.c. (double cotton covered). 208 turns of this wire will wind in one layer $16\frac{5}{8}''$ long, and 1750 turns require between 8 and 9 layers $= 0''.70$ depth.

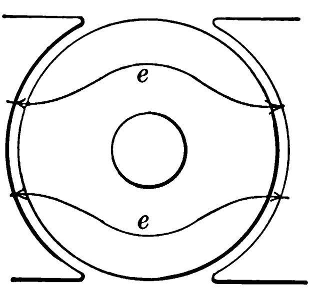


Fig. 68.

It is usual to allow a 10 per cent margin in the field resistance of shunt machines to allow for connections and the insertion of a variable resistance or "hand regulator." Making this allowance, $R = 22.5$, $\overline{cm} = \frac{11{,}970 \times 10.6}{22.5} = 5639$, which is a little greater than the area of a number 13 B. and S. wire, and the windings can be made with it. Number 13 B. and S. wire has a covered diameter of 88 mils d.c.c., and

189 turns will wind in a layer $16\frac{5}{8}''$ long, while 1750 turns require 9 layers and 49 turns over. The extra turns may be dropped, and the depth of winding becomes $0''.79$. The mean length of the winding per turn is $3'.25$, and the total length is $11{,}060'$. The total resistance is, therefore, 22.18 ohms, which is very close to that assumed, and is therefore satisfactory.

It is usual to use a practical check, as in armature wire, based upon the current in the field wire per unit cross-section. On account of the cooling influence of rotation on the armature, the *current density* in armature conductors can safely be more than double that in field conductors, therefore about 900 to 1000 circular mils per ampere is a good limit in field windings. The area of a number 13 B. and S. wire is 51,780 circular mils, which equals 1036 circular mils per ampere. We are therefore allowing ample cross-section when number 13 B. and S. wire is used. There is a disadvantage, however, in using too small a density of current in the wire, as it requires an increased total amount of copper in the windings. (Compare later pages.)

A series winding is determined as follows: The total turns $= \frac{17{,}500}{200} = 88$, or 44 per core. The approximate length of the mean turn is $3'.25$, and total length of wire $286'$. $R_f = \frac{125 \times .025}{200} = .0156$ ohms.

$\overline{cm} = \frac{286 \times 10.6}{.0156} = 200{,}000$ circular mils, and the conductor is therefore 2 number 0 B. and S. wires wound in parallel, or better, 3 number 3 B. W. G. wires wound in parallel. Number 3 B. W. G. wire has a covered

diameter of about 280 mils d.c.c., and 59 turns will wind in a layer 16⅝″ long. There are 132 turns required per core, which make 2 layers and 14 turns over, per core. Three layers deep is .84 inch, which is practically the depth assumed. Number 3 B. W. G. wire contains 67,000 circular mills in its section, hence 201,000 circular mils carries 200 amperes = 1030 cir. mils per ampere, which is satisfactory.

In all these calculations the results have not been corrected in order to provide for the pressure or current lost in the armature and fields. In a shunt wound machine, the speed can be increased two or three per cent if necessary to make up the armature loss, and the field loss merely serves to place a small additional burden upon the carrying capacity of the armature conductors. In a series machine the armature and field losses are both provided for by increasing the speed the requisite amount. It would be more scientific, but frequently less convenient, to exactly allow for these losses in making the original calculations.

Where the field cores are divided as in consequent pole machines, it is evident that the ampere turns on each divided core must equal the number required on an undivided core used for the same work. If the armature and air space be the same in the two forms of field shown in Figs. 69 and 70, it is evident that P_{a+a} is the same. To do the same work with the armatures, N_a must be equal for the two. Omitting a consideration of leakage, the same number of lines $(=N_f)$ must pass through the total section of the cores and $\frac{1}{2}N_f$ will then pass through

each core of Fig. 69. As B_f is alike in the two cases, and N_f is the same, the area of each core in Fig. 69 must become half as great as the area of the core in Fig. 70. Hence $P_{f'} = 2P_f$, where $P_{f'}$ is the reluctance

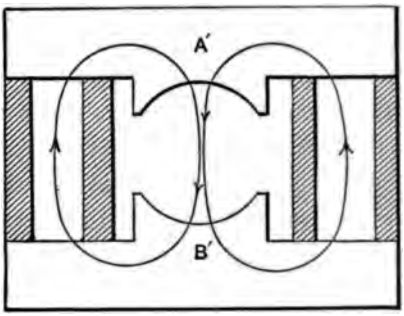


Fig. 69.

from A' to B' through either core, and P_f is reluctance from A to B through the core. But ampere turns $= \frac{\text{reluctance} \times \text{lines}}{1.25} = \frac{P_f N_f}{1.25} = \frac{P_{f'} \cdot \frac{1}{2} N_f}{1.25}$. It is evident that the weight of iron can be made alike in the two forms, while the *length* of wire required is in the ratio of $2 : \sqrt{2}$. The value of R_f should be the same in the two types, if the field loss be the same, hence the area of the wire must be directly as its length, and the weight of the copper is in the ratio of $2 : 1$.

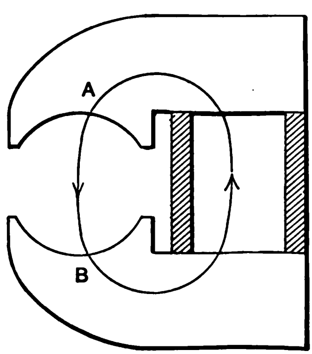


Fig. 70.

In this discussion leakage has been neglected, but

evidently the same ratios are true if v be equal in the two cases, as for the Edison type and the consequent pole type. Where v is greater, as when the consequent pole type is compared with machines using the keeper for a bed-plate, the consequent pole machine is likely to prove of proportionally greater weight, and the ratio of the weights of copper is also larger.

If one pound of copper costs eight times as much as one pound of iron in the completed machine, each pound of copper displaced by the use of less than eight pounds additional iron, effects an economy in the cost of the dynamo. Iron added to a machine frame displaces copper when it reduces the magnetic resistance of the machine. Iron which is not useful in reducing magnetic resistance, or in contributing mechanical strength or stability, is a positive disadvantage. It adds to the weight of the machine, and therefore to its cost in manufacture and handling, without making compensation. It also increases the surface from which leakage lines can emerge. Where iron is properly disposed, and therefore displaces an equivalent amount of copper, economy is also effected, because the iron can usually be handled during the processes of manufacture at less cost than can the copper.

These facts lead to machines with magnetic circuits which are carefully designed to make proper use of a maximum weight of iron. In the best types of ordinary commercial machines, the development of the magnetic circuit has reduced the magnetic resistance of the iron part of the circuit to from 10 to 30 per cent of that of the air space. Further increase of iron can therefore

give little economy. For this reason there can be little advantage in increasing the comparative weight of consequent pole machines even when compared with forms in which v is considerably smaller. To bring the single and double horseshoe types to an industrial equality, copper must be economized in the double horseshoe type. The ratio of exposed surface of field core in the double and single type is evidently greater than unity. If the total watts lost in the field windings is the same in each case, the exposed surface per watt is greater in the double horseshoe field, and the density of current in the windings can therefore be safely greater. This decreases the weight of copper in the same ratio as the increase of density, but if carried to its ultimate safe limit, the ratio of weights of copper still remains considerably greater than unity. In order that the ratio may be reduced still further, the watts lost in the field must be increased. By a judicious increase in both the C^2R loss and the density of current in the field windings, as compared with the single horseshoe type, the consequent pole machine can be made commercially satisfactory without injury to the usefulness of the machine.

It has been frequently stated that the effect of joints in the iron of the magnetic circuit is to cause a material increase in the reluctance, but this has been disproved by Ewing's experiments, which show that the effect of a scraped joint is equivalent to the introduction of an air gap with a length of from three to four thousandths of a centimeter. As the joints in dynamo frames are usually smooth planed or turned,

their equivalent air gap probably does not exceed five thousandths of a centimeter, when the joints are clean and bolted tightly together. Four joints in the magnetic circuit, therefore, may make an equivalent gap of two hundredths of a centimeter, which is not within the limit of error in measuring the armature air space.

The weight of copper on any machine can evidently be varied by changing the density of current in the wire. In the example above, if number 15 B.W.G. wire were used on the field, the dynamo would be more satisfactory in some respects, but the depth of winding would be increased, and therefore the volume or weight of copper would be greater.

As the best machines are near the limit of economy in the proportions of the iron part of the magnetic circuit, it is evident that the air space should hereafter receive the careful and particular attention of designers.

In attacking the problem of the air space, it is necessary to consider the question of armature reactions. This brings into consideration the forms of pole pieces, magnetic and electric balance of armatures, and the prevention of sparking. On the whole, the influence of the air space on armature reactions and on output is quite obscure. Deprez, Swinburne, and Ryan have endeavored to show that the magnitude of the air space should be dependent on the armature winding and the strength of field. On the other hand, Mordey, Kapp, C. E. L. Brown, Crocker, Anthony, and others, have successfully decreased the air space to a very small magnitude. The first point to be discussed covers the methods of decreasing the air space and the

effect produced on the capacity of dynamos. The first armature, made by Pacinotti, was toothed, each coil lying in a notch; but when large machines were built, it was found that the Pacinotti teeth caused trouble from sparking, and, therefore, smooth armature cores have been generally used. The toothed cores have two decided advantages:

1. The coils are protected from mechanical injury by the teeth between which they lie, and the direct mechanical bearing of the coils against the teeth materially decreases the class of armature troubles that are primarily due to the armature wires chafing against each other, during the rapid changes of "magnetic drag" through which they pass.

2. The clearance between the armature teeth (iron) and the pole pieces (iron) can be reduced to the least distance compatible with mechanical safety. Hence the magnetic resistance P_{a+a} is materially reduced. Improperly designed teeth are likely to cause vicious sparking, and excessive heating in the armature core and the pole pieces, hence their disrepute with the earlier designers. If the teeth do not cover a large part of the armature surface, the lines of force become tufted in front of each tooth, as shown in Fig. 71. The passage of these tufts across the face and through the body of the pole piece evidently must cause foucault currents, which are dissipated as heat. The heating of the armature core and the sparking at the brushes are probably due to the sudden formation and disruption of the

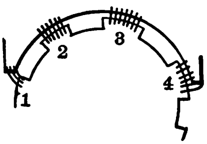


Fig. 71.

tufts, when teeth approach, and recede from, the edges of the pole pieces. The heating of the pole pieces can be prevented by making them laminated. Examples: Perrett motor, United States motor. In small motors, the sparking is so small that it may be overlooked to a large extent, and a satisfactory machine may therefore be obtained without additional precautions. In larger machines, the causes of irregular action must be removed in order to obtain satisfaction. This can be done by making the magnetic surface of the armature continuous, provided the pole pieces are properly formed. In old machines with toothed armatures, which are apparently incurably addicted to the vice of bad sparking, a more or less complete reform can frequently be effected by banding the armature with thin iron wire. This band serves to give a fairly uniform magnetic surface, and thus suppresses tufting.

In designing a new armature, a practically continuous magnetic surface can be gained by three methods:

1. Radial teeth very close together.

2. Trapezoidal or T-shaped teeth.

3. Armature conductors embedded below surface in holes or filled slots.

Each of these constructions has been used with excellent results.

Toothing or embedding evidently increases the magnetic conductivity from pole face to pole face, but it is evident that the total cross-section of the teeth under a pole face at one time cannot be equal to the full cross-section of the armature core. The teeth are therefore likely to be quite highly saturated, and the gain in mag-

netic conductivity is not proportional to the reduction of the air space. For instance, if the outside diameter (over the bands) of the 25 K.W. armature referred to on page 113, be $12\frac{1}{4}''$, and the mechanical clearance $\frac{1}{8}''$, the bore of the pole piece will thus be $12\frac{1}{2}''$ in diameter, and the double length of the air space $1''$. If the wires be placed in slots, 4 wires deep and 2 wires wide (Fig. 72), one-half the armature surface would be occupied by slots, leaving the other half for teeth. The air space can now be reduced, as far as mechanical safety will admit, to, say, $\frac{5}{16}''$ (.31), including bands, and the depth of the slots may be about $0''.7$. The reluctance between pole face and armature core is made up of $0''.155$ of air and $0''.7$, which is one-half air and one-half iron. If it be assumed that all the lines of force pass through the teeth, then B in the teeth is about 20,000 ($B_a = 10{,}000$), and μ for the teeth is about 30. Hence the reluctance of the air space becomes less than 0.4 of its value for the smooth core. When the conductors are dimensioned with special reference to a reduction of the space occupied by wire, the value of P_{a+a} for a toothed armature can be reduced to a value as small as one-fourth its value when the armature core is smooth. If the armature is worked at a lower magnetization, toothing has a more marked effect. Thus, if B_a be 8000 in the case above, B for the teeth is about 16,000, μ becomes 300, and the total air space reluctance is less than $\frac{1}{3}$ its value for the smooth core.

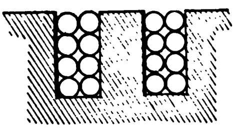


Fig. 72.

As the air space reluctance of properly designed two-

pole dynamos with smooth armature cores must be from 70 to 90 per cent of the total reluctance of the magnetic circuit, the reduction of reluctance by toothing produces a marked economy in magnetizing force, and therefore in copper. Examples: Crocker-Wheeler motor, United States motor.

Since all wires in which a current flows are linked by lines of force of their own, the current in the conductors of an armature sets up a magnetization which is distinct from the field magnetization, and the number of effective ampere turns on an armature is equal to $\frac{1}{2}$ the current multiplied by the number of conductors on $\frac{1}{2}$ the armature ($\frac{1}{2}C \times \frac{1}{2}S = \frac{1}{4}\overline{SC}$). When the commutating* plane coincides with the normal* plane of a dynamo, the magnetizations due to the field magnets and to the armature current, tend to set up poles in the armature 90° apart. This is the condition of a stationary armature to which a current is given on the normal plane, as shown in Fig. 73. As lines of force of like direction tend to crowd together, a resultant magnetization of the armature is caused, which lies between the two components. Assuming the two impressed magnetizations to be proportional to the ampere turns, respectively, on field and armature (which is not greatly in error, see Chap. VI.), they may be represented as two

* The neutral plane may be defined as a plane which passes through the centre line of the armature perpendicular to the lines of force.

The normal plane is one which is perpendicular to the line joining the centres of the pole pieces.

The commutating plane is a plane which passes through the centre line of the armature and cuts the commutator at the points of contact of the brushes.

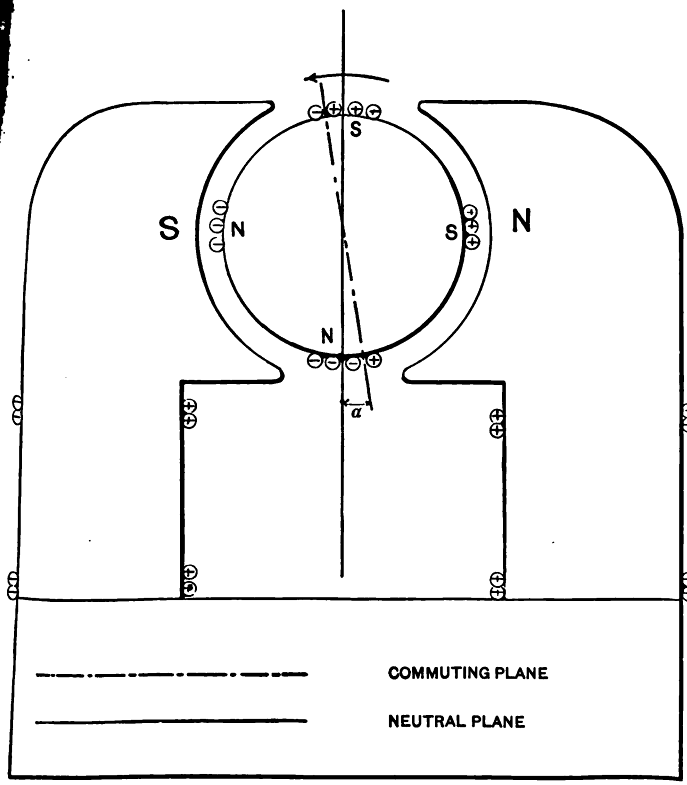


Fig. 73.

forces subject to the triangle of forces, and their resultai will be represented in magnitude and direction by th diagonal of a parallelogram, the two adjacent sides o which represent the two forces.

Suppose *OF* in Fig. 74 is the magnetization due to the field when there is no current in the armature. If a current be passed through the armature, the magnetization due to it may be represented by *OA*, and the resultant of *OA* and *OF* by *OR*. The increase in the total length of the lines of force which pass through the armature and air space, due to the armature magnetization, makes it necessary to increase the magnetizing force in order that the field may be kept constant, *i.e.* the field ampere turns must be increased from *OF* to *OR*. This relation is not exactly true on account of the different values of the reluctance in the path of the lines of force set up by the two windings *OA* and *OF* (see Chap. VI.), and also on account of the bend in the curve of magnetization, but it is sufficiently exact to be of material use in the practical design of many types of dynamos.

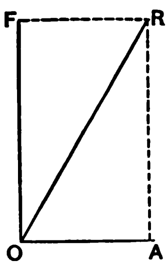


Fig. 74.

When the brushes have a positive or forward **Lead*** (see Fig. 73), which is true to a greater or less extent in all dynamos, the ampere turns on the armature can be considered as causing two magnetizations which are at right angles to each other. Those turns *within*

* When the commutating plane does not coincide with the normal plane the brushes are said to have a *lead*, which may be either positive or negative with the direction of the armature rotation.

the *double angle of lead* α, are directly opposed to the impressed field magnetization, and the rest act to skew the field lines, as explained above. In a motor the same relation exists. The relative direction of field and armature magnetizations is reversed for a given direction of rotation, but the lead is negative or trailing, and the turns within the double angle of lead oppose the field magnetization as before. The turns opposing the field magnetization may be called the **Back turns,** and the others the **Cross turns.** The parallelogram of forces previously explained should be taken between the field ampere turns, calculated with an open external circuit, and the cross turns. In an ordinary machine, when the current causing OC cross turns (Fig. 75) flows through the armature, the field ampere turns must be approximately proportional to $OR+$ the back turns, in order to give constant pressure at constant speed.

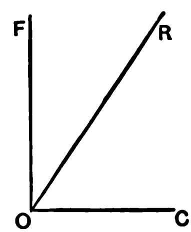


Fig. 75.

The following examples show the application of this reasoning to dynamos and motors of various forms.

Double horseshoe type, 110 volts, 80 amperes. Shunt field ampere turns $=7800$; series field ampere turns at full load $=720$, making the total field ampere turns at full load $=8520$. Armature ampere turns at full load $=5760$, $\alpha=15^\circ$, back turns $=480$, cross turns $=5280$. To compensate for the loss of pressure due to armature and series field resistance, about 400 ampere turns are needed. By calculation, $OR=7800$. Adding 400 ampere turns and 480 ampere turns to compensate

the loss of pressure and the back turns, makes the total calculated ampere turns on field, at full load, 8680 and the increase required for compensating the effect of the load 880, which is not wide of the mark.

Single horseshoe type, 125 volts, 80 amperes. Shunt field ampere turns at no load = 9470, at full load = 13,440. Armature ampere turns, full load 4000. $\alpha = 0$.

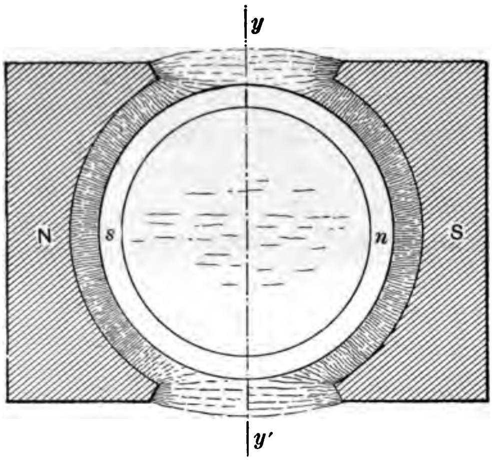


Fig. 76 a.

Calculated field ampere turns, full load, 12,400. To compensate the loss of volts due to the armature resistance requires, at full load, an additional 600 ampere turns, more or less; making the total calculated ampere turns, at full load, 13,000, which is very close to the number experimentally determined.

Inverted single horseshoe type, 112 volts, 24 amperes. Shunt field ampere turns at no load = 2600. Shunt field ampere turns at full load = 3200, of which 200 are

required to compensate loss of pressure due to armature resistance. $\alpha = 0$. Armature ampere turns at full load $= 2300$. Calculated increase in ampere turns at full load, allowing for the C^2R loss, is 1000.

Double horseshoe type, 400 volts, 150 amperes. Shunt field ampere turns $= 32,700$. Series field ampere turns at full load $= 1950$. Ampere turns required to compensate C^2R loss in armature at full load = about 700. $\alpha = 0$. Armature ampere turns at full load $= 14,500$. Calculated increase in ampere turns at full load, allowing for C^2R loss, is 3800.

Single horseshoe type, 500 volts, 100 amperes. Shunt field ampere turns $= 21,800$. Series field ampere turns at full load $= 4450$. Armature ampere turns at full load $= 8700$. $2\alpha = 15°$. Back turns $= 700$, and cross turns $= 8000$. Compensation for armature C^2R loss and over-compounding = about 2000. Calculated series field ampere turns at full load, allowing for C^2R loss and over-compounding $= 4100$.

Inverted single horseshoe type, 110 volts, 30 amperes. Shunt field ampere turns $= 5200$. Series field ampere turns at full load $= 2400$. Armature ampere turns at full load $= 5400$. $\alpha = 0$. Compensation for armature C^2R loss = about 1000. Calculated series field ampere turns at full load, allowing for C^2R loss, $= 3300$.

It will be noticed that the calculated increase of field turns required to compensate the effect of armature reaction is usually greater than the actual operation of the machine required. In other words, the armature reactions have a smaller effect than is assumed in the application of the parallelogram of forces. This plan

seems to have been originally applied by Dobrowolsky to dynamos in which the armature wires were embedded in the iron of the core, which therefore had a magnetic circuit for the lines of force due to the armature wires, of comparatively small reluctance. In such armatures the reactions are large. For ordinary armatures, where the wires are on the surface of a drum, if OC is taken as from $\frac{1}{2}$ to $\frac{3}{4}$ of the cross turns, as suggested by Kapp, the result comes closer to the truth. Seven-tenths may

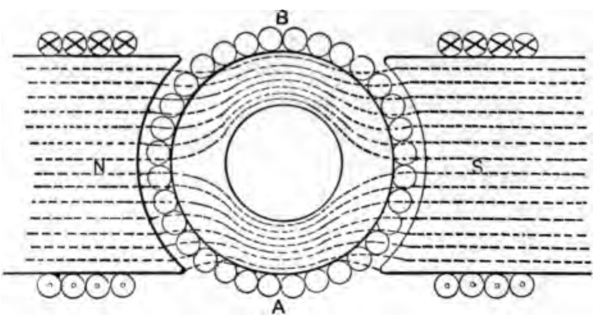


Fig. 76.

Lines of force due to field coils; no current in armature.

be taken to represent a fair average to be used in general designing. For example, in the case of the last machine given above, if the resultant of the field ampere turns at no load, and 0.7 of the armature ampere turns at full load, be calculated, the number of additional field ampere turns at full load is shown to be practically 2350, which is almost exactly the number shown by experiment to be necessary in maintaining a constant pressure. Designers of some types of machines often calculate the series field ampere turns

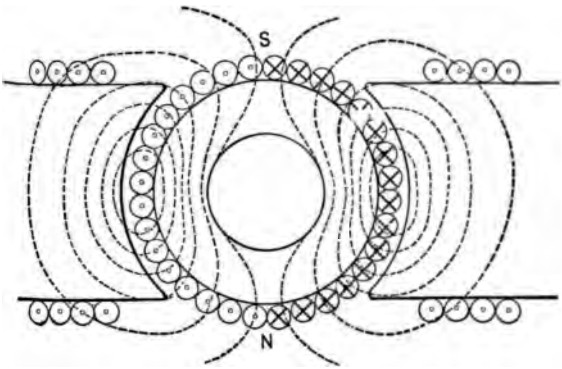


Fig. 77.

Lines of force due to armature; no current in field coils; current brought to armature on neutral plane *NS*.

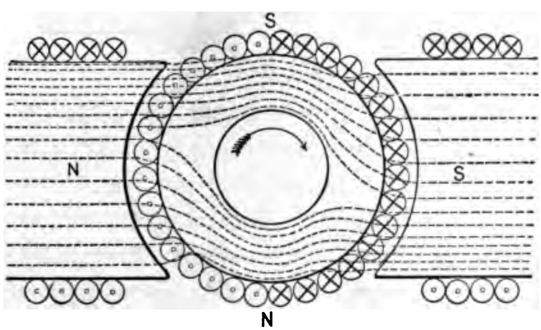


Fig. 78.

Lines of force due to the resultant effect of current in armature and d windings; current brought to armature on neutral plane *NS*.

as a fixed ratio of the ampere turns on the armature. Esson gives the ratio which the compensating or series ampere turns should bear to the total armature ampere turns as two-thirds for machines with Gramme armatures.

The cross-turns on the armature tend to change the distribution of lines of force on the pole faces, skewing them over from the *entering tip* to the *trailing tip* of the pole pieces in the case of a generator, and in the opposite direction in the case of a motor. In order

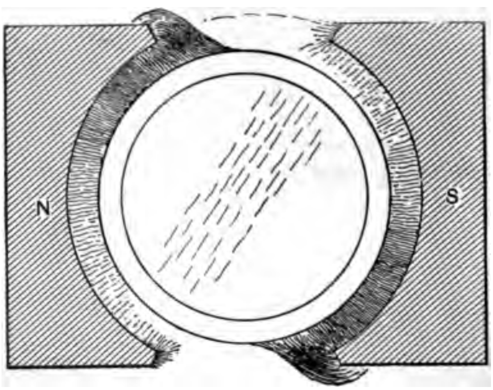


Fig. 78a.

that a machine may give a maximum output with the least sparking, it is necessary for the brushes to be set at the points where the coils do not cut the lines of force of a strong field. Hence, when the resultant direction of the lines of force passing through the armature is changed, the commutating plane must also be changed. Each advance of the commutating plane evidently causes an advance in the position of the poles in the core due to the ampere turns on the armature, and causes a greater skewing of the lines of force (see Figs. 6 to 79).

If no other action came into play, it would be impossible for the forward movement of the brushes to catch up with the neutral plane, but in practice, as the commutating position is advanced, the lines of force are crowded, or piled up, in the trailing pole tips, until further skewing of their path through the armature is impossible, and the commutating plane overtakes the

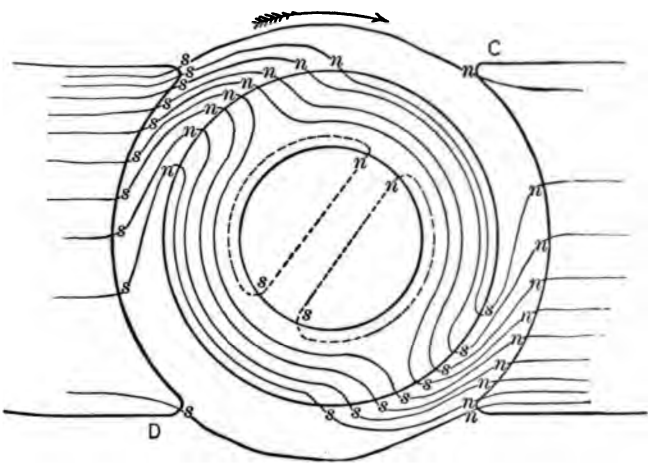


Fig. 79.

The effect on resultant lines of force caused by moving poles in armature through bringing current to it on commutating plane *CD*.

neutral plane. The result of the lines of force skewing or piling up in the trailing pole tips is to give a sensitive commutating plane, and therefore a lead which must be varied with all changes of load. It is also likely to cause sparking, which cannot be entirely suppressed. It therefore is desirable to so form the pole pieces that the piling up of lines of force cannot

occur. The first point to be gained is a *stiff field*, *i.e.* one which in itself has considerable stability. This requires a strong field, with a uniform distribution over the pole faces. By a proper design of the magnetic circuit and of the magnetizing coils it is easy to gain a strong field. To get a fairly uniform distribution over the pole faces, even when the armature is at rest, requires careful designing of the pole pieces. All corners on the pole pieces that are presented to the armature must be carefully rounded off, and the ends of the pole pieces must be bell mouthed, as shown in Fig. 80.

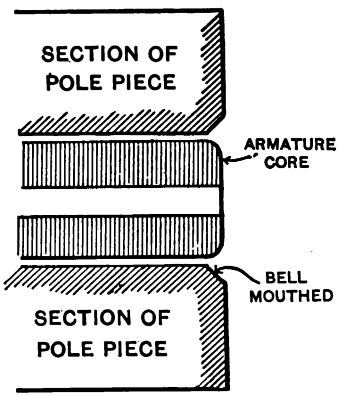


Fig. 80.

To avoid the piling up of lines of force at the tip, it is necessary to increase the magnetic resistance of the air space (or its equivalent) at the sides of the pole pieces, and under the tips. This can be done either by thinning down the pole pieces or by increasing the air space under the corners. The latter plan, which is shown at the left hand of Fig. 81, is usually most satisfactory, as the corners can be well rounded off, and the reluctance toward the sides is not dependent upon the saturation of the pole tips. The form shown in the right hand of Fig. 81, even with the armature idle, is likely to give a distribution that is thinnest at the middle of the face and strongest at the tips. It is evidently dependent

upon the saturation of the trailing tips "*a*" to hold the magnetic lines in position, and the field is not *stiff*, but it may be entirely satisfactory when the leakage between the pole tips is sufficient to keep them always saturated. The first form causes the lines of force to be thinnest near the edges, when the armature is idle, on account of the increased air space under the tips. The field is therefore more likely to be *stiff*. The stability of the field and the value of v also depend on the *angular embrace* (angle β, Fig. 81) of the pole pieces. A large embrace brings the tips close together and increases v to an excessive value, while a small embrace decreases the cross-section of the air space and thus increases its reluctance. In the best dynamos the angular embrace usually varies from 120° to 140°, and its exact value must be determined by the judgment of the designer.

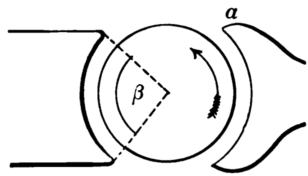


Fig. 81.

The form of the pole tips has a decided influence on the lead, and the obtaining of sparkless commutation. While an armature coil approaches the positive brush, for instance, one-half the total current flows through it towards the brush. When the coil reaches the brush, it is short-circuited by the toe of the brush resting across the two commutator strips to which the coil is connected. When the coil has passed beyond the brush, half the total current again flows through it towards the brush, but the *direction of the current through the coil is reversed.*

If commutation be effected in a field which tends to uphold the current in the short-circuited coil, a large current may flow, with the result of excessive heating in the armature. This current must be forcibly reversed as the coil leaves the brush, and therefore to prevent sparking, the commutating plane must be in angular advance (negative for a motor) of the neutral plane, an amount which is sufficient for the short-circuited coil to be influenced by the field of the entering pole tip. This field can be made of such strength that, during short circuit, the current in the coil is reduced to zero, reversed, and brought up to its normal value. There can then be no undue heating, or sparking when the coil is again introduced into the circuit. In order that the current in the short-circuited coil may be influenced by a weak field, thus making a small lead possible, the self-induction, and therefore the number of turns in each coil, must be small.

To easily effect the proper reversal of current in the short-circuited coil, the pole tips are often extended in the middle (Fig. 82). The extension should be rather thin on the edge, so that the field under it is small and fairly uniform. The brushes may then have considerable range of position under a given load, without causing sparking. To avoid a large lead, and consequent back turns, the elongated tips probably should come within 15° or 20° of the normal plane, while the polar embrace measured to the points *A* probably should not exceed 130°. (Compare *Transactions American Institute of Electrical Engineers*, Vol. 7, p. 218.)

It must always be remembered that sparking can be

entirely suppressed only when the armature is in good magnetic and electric balance. An armature is in magnetic balance when coils at the same angular distance from the commutating plane, but on opposite sides of it, are threaded by the same number of lines of force, and when the coils short-circuited by the two brushes have the same angular position with reference to the neutral plane. An armature is in electric balance when all the coils have the same electrical resistance.

A Gramme armature is usually in good electric balance, but is likely to be out of magnetic balance when

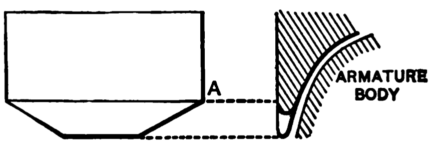


Fig. 82.

used in single horseshoe fields, unless the latter are very carefully designed. A horizontally wound Siemens armature is usually out of electric balance, but in magnetic balance. A vertically wound Siemens armature may be so wound that it is exactly in both magnetic and electric balance.

The form and cross-section of the pole pieces have another element requiring attention in the case of single horseshoe frames. If the pole pieces are tapered at the ends, as at *CC* in Fig. 83, considerably more than one-half of the lines of force may enter and leave the armature upon the keeper side of the line *AB*. In this case there will be a resultant pull, due to the tension

of the lines of force, which will crowd the armature against the keeper side of the bearings and may cause severe heating. In the inverted single horseshoe the weight of the armature will partially or wholly counteract this pull, and in the ordinary type the armature may be placed eccentrically in the bore of the pole pieces, so that the air space is longer below. Other devices for making the reluctance symmetrical have also been used, but the best plan is to make the ends of the pole pieces of a sufficiently large cross-section to cause the field to be naturally symmetrical with regard to the plane AB.

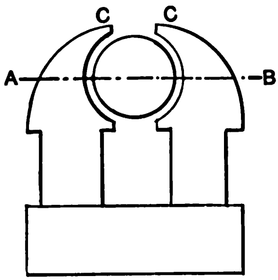


Fig. 83.

CHAPTER VI.

COMPENSATION FOR CROSS-TURNS, AND THE EFFECT OF BRUSH CONTACT.

WITH the magnetic effect of the armature coils compensated, the difficulties encountered in obtaining a sparkless collection on a plane of commutation which is stationary at all loads, would almost vanish. Many plans for effecting the compensation have been advanced, but they have usually been of an impracticable nature. Consequently the methods, already discussed (Chap. V.), for reducing the effect of the evil to a minimum, have been generally adopted to the practical exclusion of schemes for its eradication. The latest device for compensating the effect of cross-magnetization has been brought to apparent success by Professor H. J. Ryan, though little practical experience in the use of his device has yet been had. If each armature coil were paralleled by an equal coil which carried an equal current in the opposite direction, it is evident that the magnetic effect of the armature coils would be neutralized. Such a neutralizing layer of coils exists in a motor generator when the motor and dynamo armature wires are wound side by side on the same core, as is sometimes done, but in an ordinary machine the neutralizing coils cannot be placed on the

armature, nor can they be placed in the air space unless the depth of the gap be increased unduly. However, a series of coils may be arranged to embrace the armature in the general plane of the lines of force, by pass-

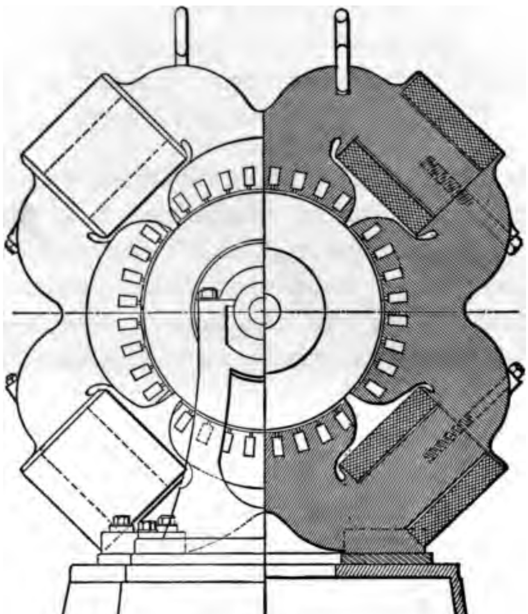


Fig. 84.

ing wires through holes or grooves made in the faces of the pole pieces (Figs. 84 and 85). If the turns of wire in the coils be equal in number to one-half the number of cross-turns and be connected in series with the external circuit of the dynamo, their magnetic effect can doubtless be made to practically neutralize the effect of the

cross-turns. If the neutralizing layer in the faces of the pole pieces be uniform over the whole of the faces, and the commutating plane remain stationary under

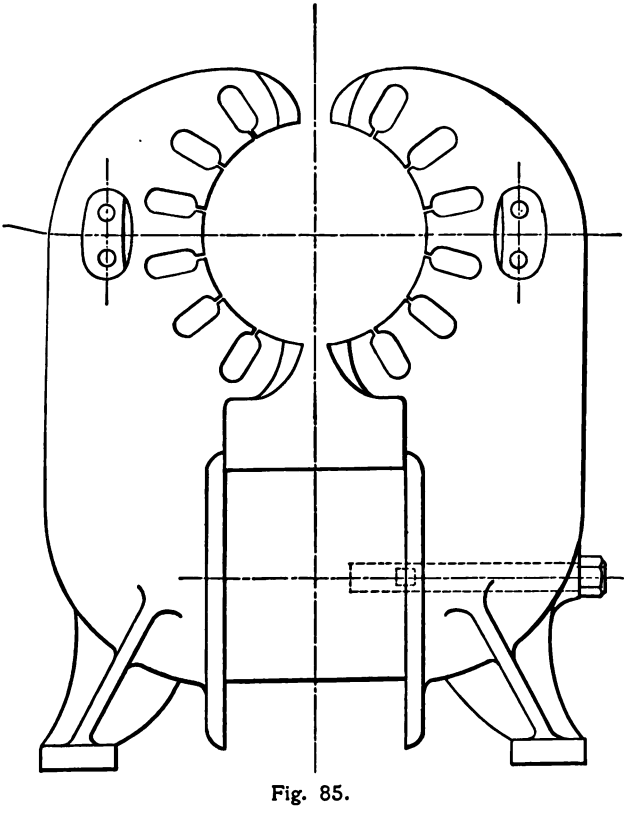


Fig. 85.

ıe entering corners of the pole pieces, the neutraliza-
on ought to be quite exact. With a lead either greater
· less, a portion of the cross-turns must remain to a
:rtain extent uncompensated, and if the lead be vari-

able, the compensation may be fairly exact at one load and less exact at others. Professor Ryan says, regarding one of the dynamos built according to his designs:

"In the performance of this machine, as well as in that of the first one that we constructed, we find that by the use of the balancing coils all cross-induction may be avoided or it may even be reversed in its effects. In the first machine the number of balancing turns was variable and it was found entirely possible to over-compensate the armature reactive effects so as to get the strong pole corners where we ordinarily would get the weakened ones. We find with the use of the balancing coils that the magnetization through the poles, air gap, and armature do not undergo a redistribution under any load; that the armature current does not alter the total magnetization through the armature at any value; that the neutral point remains constant; that the diameter of commutation with metallic brushes changes only by the small amount necessary to balance the self-induction of the commutated section by the field in which it is moving; that regulation for constant current may be effected without change of the diameter of commutation by varying the magnetization through any limits; that by proper designing, the output for a given weight of the completed machine may be quite largely increased over that which is realized in the common practice of to-day; and that the air gap may be made as small as mechanical requirements will permit without changing the performance of the machine, thus enabling one to realize practically the advantages of differential excitation that is utilized

so successfully in the modern alternate current transformer."

Figure 84 shows a half-elevation and half-section of a small four-pole machine arranged for Ryan coils, and Fig. 85 shows the elevation of a two-pole machine similarly arranged. The first machine is a shunt wound dynamo of 16,000 watts capacity designed to run at 400 revolutions per minute; the second machine is a series motor designed to run at 1960 revolutions per minute under a constant pressure of 110 volts and to develop two horse-power. The effect of the compensating coils is excellently shown in the series of figures 86 to 92, which represent magnetic distribution curves

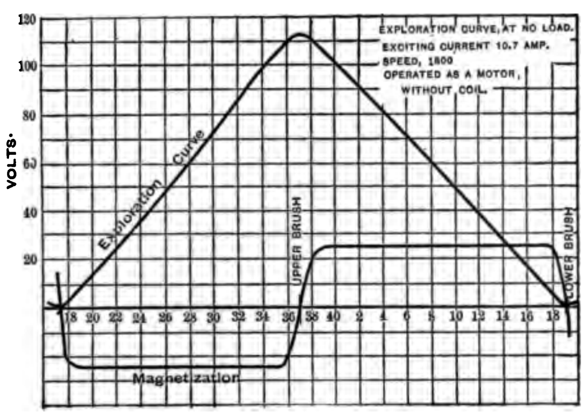


Fig. 86.

taken by the pilot-brush method (page 209), while the machine of Fig. 85 was operated under various conditions. The effect of the compensating coil is made so evident by the figures that comment is unnecessary.

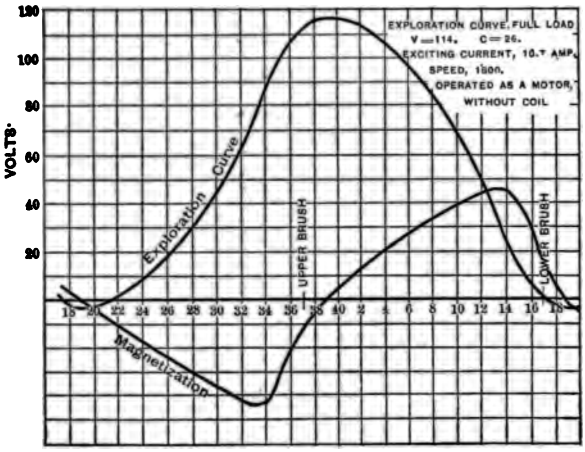

EXPLORATION CURVE, FULL LOAD
V = 114. C = 26.
EXCITING CURRENT, 10.7 AMP.
SPEED, 1800.
OPERATED AS A MOTOR,
WITHOUT COIL
VOLTS
120
100
80
60
40
20
Exploration Curve
Magnetization
UPPER BRUSH
LOWER BRUSH
18 20 22 24 26 28 30 32 34 36 38 40 2 4 6 8 10 12 14 16 18

Fig. 87.

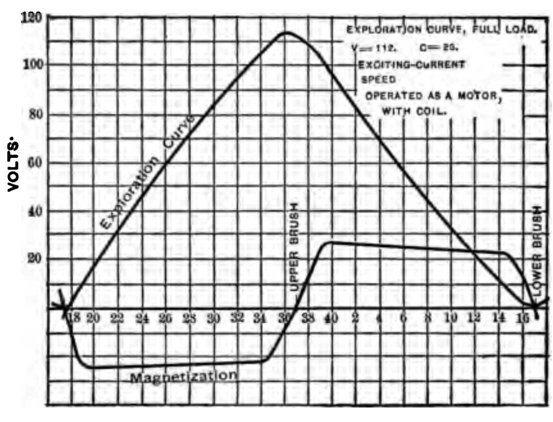

EXPLORATION CURVE, FULL LOAD.
V = 112. C = 26.
EXCITING-CURRENT
SPEED
OPERATED AS A MOTOR,
WITH COIL.
VOLTS
120
100
80
60
40
20
Exploration Curve
Magnetization
UPPER BRUSH
LOWER BRUSH
18 20 22 24 26 28 30 32 34 36 38 40 2 4 6 8 10 12 14 16

Fig. 88.

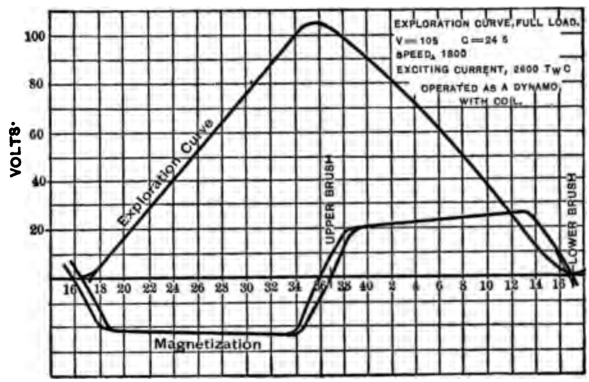

EXPLORATION CURVE, FULL LOAD.
V=105 C=24 5
SPEED, 1800
EXCITING CURRENT, 2600 TwC
OPERATED AS A DYNAMO, WITH COIL.
VOLTS.
Exploration Curve
Magnetization
UPPER BRUSH
LOWER BRUSH

Fig. 89.

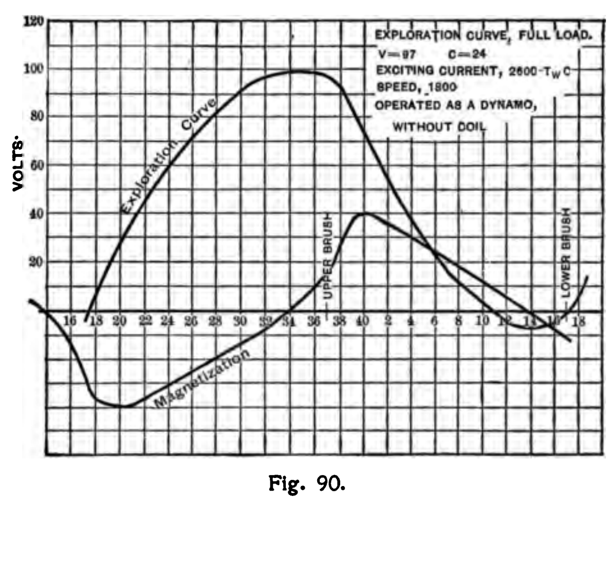

EXPLORATION CURVE, FULL LOAD.
V=97 C=24
EXCITING CURRENT, 2600 TwC
SPEED, 1800
OPERATED AS A DYNAMO, WITHOUT COIL
VOLTS.
Exploration Curve
Magnetization
UPPER BRUSH
LOWER BRUSH

Fig. 90.

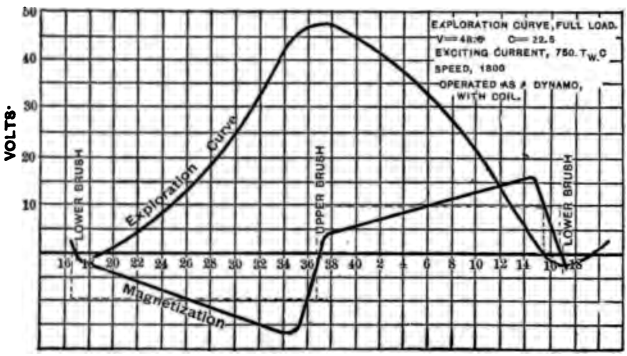

EXPLORATION CURVE, FULL LOAD.
C = 22.5
EXCITING CURRENT, 750. T_W.C
SPEED, 1800
OPERATED AS A DYNAMO, WITH COIL.
VOLTS
LOWER BRUSH
UPPER BRUSH
LOWER BRUSH
Exploration Curve
Magnetization

Fig. 91.

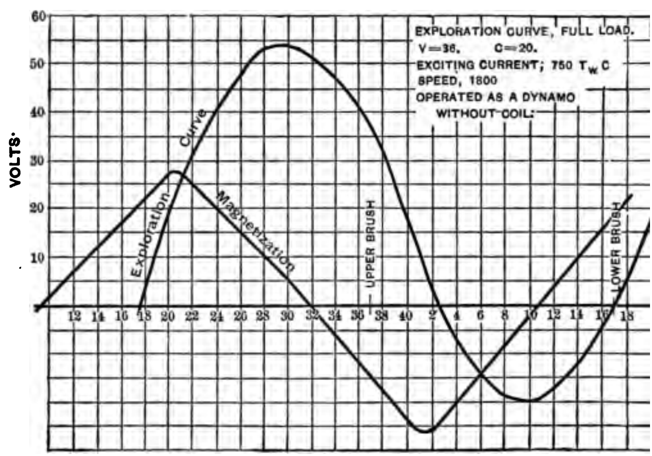

EXPLORATION CURVE, FULL LOAD.
V = 36. C = 20.
EXCITING CURRENT; 750 T_W C
SPEED, 1800
OPERATED AS A DYNAMO WITHOUT COIL.
VOLTS
UPPER BRUSH
LOWER BRUSH
Exploration Curve
Magnetization

Fig. 92.

A scheme similar to Professor Ryan's has been suggested by Fischer-Hinnen (*vide Elektrotechnische Zeitschrift*, 1893, and *London Electrician*, Vol. 31). This is shown in Fig. 93, and consists of a single coil of wire wound in a groove in the face of the pole pieces and as nearly as possible perpendicular to the neutral plane. The magnetizing effect is in such a direction as to oppose the effect of the cross-turns of the armature, and if the turns of the coil are in proper number and connected in the main circuit, the effect of the cross-turns may be practically neutralized.

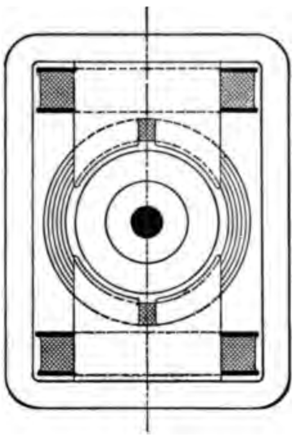


Fig. 93.

Several other methods for balancing or neutralizing the effect of cross-turns have been used to a limited extent. At one time it was not unusual to wind the series or compounding coils of a machine upon the ends of the field cores close to the entering or weakened pole corners. In double horseshoe types, with an excessive leakage coefficient, the arrangement shown in Fig. 94 might be of some service, but in machines of the best design the path of the lines of force is not materially changed by changing the positions on the core which the windings occupy. Hence, irregular or eccentric winding fails of its purpose (which is to strengthen the pole corners near it), while it increases

the cost of construction. Strengthening the weakened pole corners might be better effected by a special pole

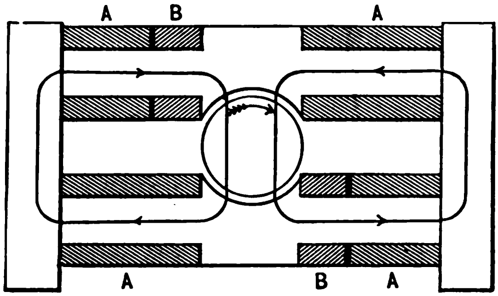


Fig. 94.
A, shunt coils; *B*, series coils.

piece winding, as shown in Fig. 95, which is quite similar to an arrangement suggested by Swinburne.

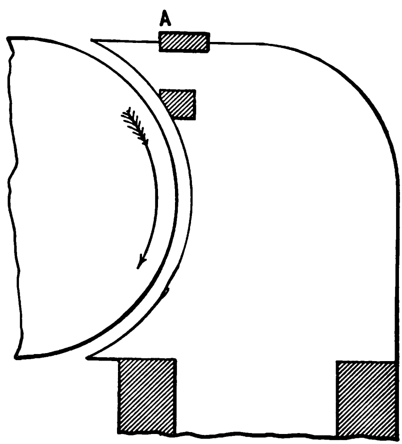


Fig. 95.
A, special pole piece coils.

The winding might be applied to both corners of each pole piece so as to strengthen one and weaken the

other. Another similar device is shown in Fig. 95*a*. Even this, however, would doubtless only partially accomplish its object, and would be an expensive construction. An advance upon the eccentric compound winding was made by Elihu Thomson, who placed his compound coils so as to directly embrace the armature, but tipped backwards so that the wires lay close to the trailing tips of the pole pieces (Fig. 96). The vertical component of the lines of force due to the tipped compound coils was expected to cause a direct balancing of a considerable portion of the cross-turns. While the expectation

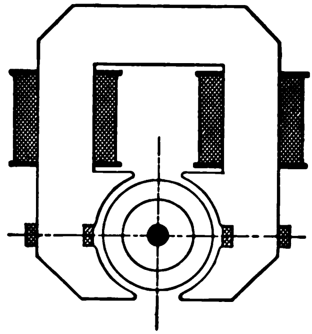


Fig. 95*a*.

A A, Tipped Series Coils.

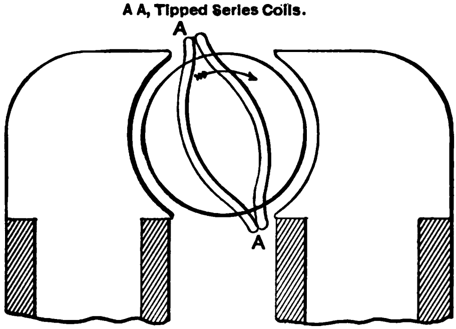


Fig. 96.

was met to a limited degree, the effect seems to be mostly due to the lines of force of the series coils directly influencing the pole corners, as shown in Fig. 97.

As the effect thus exerted on the pole corners does i
interfere with the true compounding function of t
coils, the arrangement seems to be somewhat adva
tageous, but not sufficiently so to secure its final ado
tion on machines with the best forms of magnet
circuits. Other devices for neutralizing the effect c
cross-turns which have been tested, but which hav
not been of such a nature as to recommend them fo
practical use, are numerous. A device that has re-

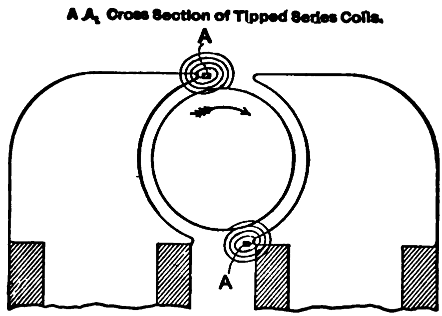


Fig. 97.

ceived considerable attention is a pair of auxiliary series magnets (Fig. 98) placed so as to influence the short-circuited coil at commutation. Such an arrangement is readily conceived, but it apparently is not possible to make it effective in operation on account of variations of lead and the irregular curvature of the curve of magnetization.

The effect of the Ryan device upon the permeability of the pole pieces is a matter of some importance. It may be said to be proved that the permeability of iron

measured along any path is dependent, to a considerable degree, upon the number of lines of force passing through the iron in any direction. In the Ryan device a magnetizing coil is placed in the face of the pole pieces, and lines of force are developed around the individual turns of the coil. These lines of force place some magnetic burden upon the iron of the pole pieces, the reluctance of which is originally increased by the spaces arranged to hold the compensating turns.

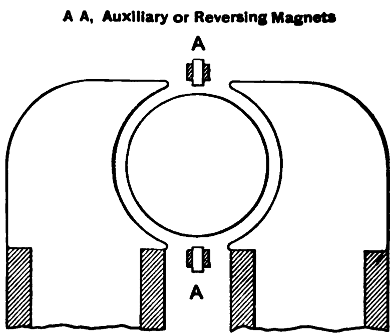


Fig. 98.

On the other hand, the compensation of armature reactions avoids the crowding of the lines of force into one corner of each pole piece, and therefore serves to decrease the actual reluctance of both the pole pieces and the air gap in the path of the effective lines of force. No doubt the Ryan device causes a considerable increase in the reluctance across the pole pieces from tip to tip, which in itself must tend to reduce the magnetic effect of the cross-turns. The strength of the field due to the cross-turns is evidently proportional

to their number divided by the reluctance of the magnetic circuit which is approximately made up of the armature core, twice the air space reluctance, and the reluctance from tip to tip of a layer of the pole pieces (this layer is probably only a few inches in depth, Figs. 99 and 100). The cross-turns on each half of the armature are * obviously equivalent to a sheet of current flowing

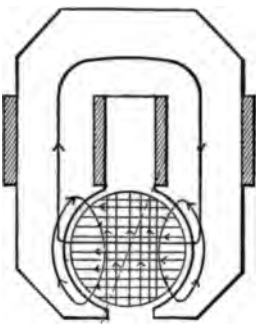


Fig. 99.

between two parallel iron surfaces of breadth λ and the distance δ apart, the total strength of current being $sc\frac{\lambda}{\pi d}$; whilst the current density per centimeter is $\frac{sc}{\pi d}=\gamma$ (λ being the length of the polar arc, δ the depth of the air gap, d the diameter of the armature, s the number of armature conductors, and c the current in each conductor). To determine the effect of the sheet of current on the induction between the two surfaces, we suppose the latter to be straightened out into a plane (Fig. 100), where AA represents the surface of the armature, PP that of the pole, CC the sheet of current. Selecting any point p on the pole face at a distance a from the centre, we find that the induction within the air space at p is due to the action of all the current elements to the right and left of the point, the integration being extended to the edges of the polar face. A current

* Kapp, on dynamos in *Practical Electrical Engineering.*

element γdy at the distance y from p produces a magnetizing force $H = \frac{1.25\,\gamma dy}{2\delta}$, and this integrated over all the elements to the right of p gives the induction through p due to the part of the current sheet that lies to the right of p. Neglecting the comparatively very small magnetic resistance of the iron part of the path of lines, this induction is $\frac{1.25\,\gamma}{2\delta}\left(\frac{\lambda}{2} - a\right)$. In a similar manner we

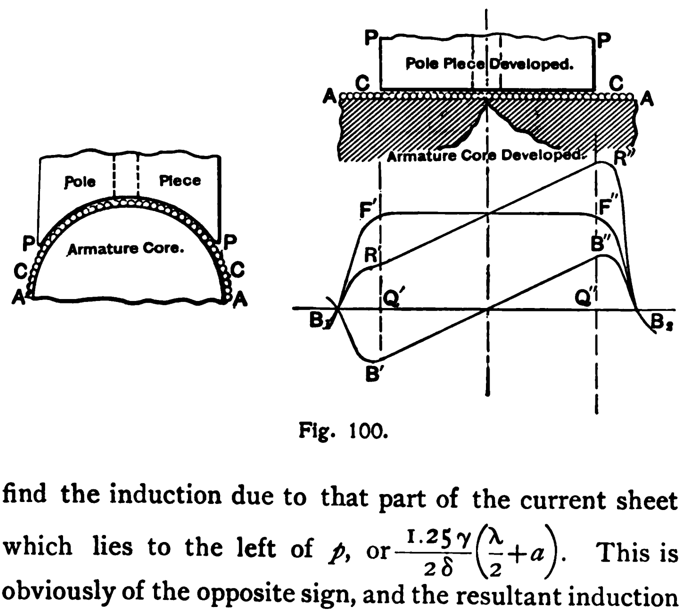


Fig. 100.

find the induction due to that part of the current sheet which lies to the left of p, or $\frac{1.25\,\gamma}{2\delta}\left(\frac{\lambda}{2} + a\right)$. This is obviously of the opposite sign, and the resultant induction is the algebraical sum of these two values; namely, $\frac{1.25\,\gamma}{2\delta} 2a$. For $a = 0$ (*i.e.* for the centre of the pole piece), the induction is zero, for $a = \frac{\lambda}{2}$ (*i.e.* for the edges of the pole piece) it is a maximum, being positive for

one and negative for the other edge, as shown by the sloping line BL. Its value is

$$\frac{1.25\gamma\lambda}{2b} = \frac{1.25}{2b} \times sc\frac{\lambda}{\pi d}.$$

This is the induction due to the armature cross-turns only, but in addition there is the induction due to the exciting coils of the field magnets, and to find the true induction within the air-space we add these two values.

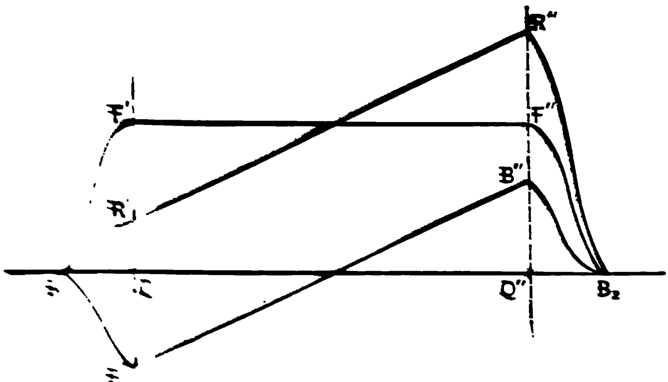


Fig. 101.

In Fig. 101 is shown the curve $B_1B'B''B_2$, representing the induction due to the cross-turns (also Fig. 100), and also the curve $B_1F'F''B_2$ representing the induction due to the field magnets. The curve $B_1R'R''B_2$ represents the resultant magnetism. The figure shows plainly that any construction which increases the reluctance in the path of the lines of force due to the cross-turns, and thus decreases the height of $Q'B'$ and $Q''B''$, must decrease the distortion of the normal field. This increase

of reluctance may be gained by uniformly increasing the depth of the air space; by making a single slot across the middle of the pole piece, as shown by the dotted lines in Fig. 100; by placing slots at intervals all the way across the face of the pole piece, as in the Ryan device (Figs 84 and 85); by arranging the pole corners so that they are continuously saturated by magnetic leakage; or by making the bore of the pole pieces elliptical, as explained on page 167. By the last two methods the reluctance in the path of the lines due to the cross-turns is materially increased without an equivalent increase of reluctance in the path of the effective lines, and since the expense of this construction is small in many types of machines, it is particularly advantageous. That any change in the depth of the air space has a marked influence on the number of lines of force developed by the cross-turns on an armature, is a matter of common observation. A maximum effect is observed in machines where the armature conductors are embedded in the iron of the armature core, when the mechanical clearance, and consequently the air space, is small. It has become a common practice to simply increase the air space of such machines by boring out the pole pieces, in order that the evil results of a variable lead and severe sparking may be reduced. This increases the reluctance of the path of useful lines of force (as well as that of the lines of force from armature turns), and necessitates an additional expenditure of energy for excitation, which in turn requires additional cost for the copper in the field magnet windings. It may be safely said that this construction simply reduces the

effect of the evil instead of striking at its root, and it causes the loss of much of the advantage of embedded armature conductors. The lamination of the pole pieces in planes parallel to the armature shaft has been tried with excellent results. The lamination required for this purpose is at right angles to that mentioned on page 154, but it might serve to reduce foucault currents in the pole pieces equally well.

It would seem, as already stated, that the mutual magnetic effect of the two windings on the double armature of a motor-generator with one armature should be zero. It is found in practice, however, that vicious sparking is a common habit of motor-generators of any considerable capacity which have the motor and dynamo windings on the same armature core, and that the commutating plane of both commutators is very sensitive. The difficulty seems to be caused by the mutual induction between the two windings, on account of which the two currents mutually interfere at commutation with the proper reversal of current in the coils which are short-circuited. On account of this difficulty it is now usual to make each motor-generator with two sets of fields and two armatures, so that it virtually consists of a separate motor and generator with their shafts coupled.

Instead of neutralizing or balancing the magnetic effect of armature cross-turns, an entirely different scheme may be used to effect commutation without sparking. If a counter-electric pressure be introduced in the short-circuited coil by external means, of such strength and during such an interval that the current in

the coil is reversed and brought up to the proper value for introduction into the circuit, commutation may be effected when the short-circuited coil is in any position, regardless of the influence of the field. The electric pressure introduced must evidently vary with the load, in order that the current in the short-circuited coil may always have the proper value when the coil comes into circuit. A very simple and apparently useful device for introducing the electric pressure in the short-cir-

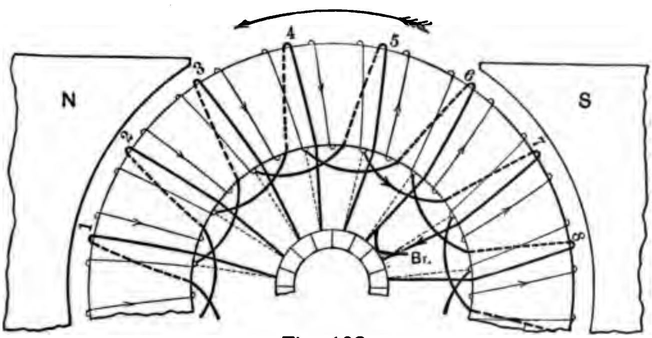


Fig. 102.

cuited coil has been proposed by W. B. Sayers. (See *Journal Institution of Electrical Engineers*, 1893; *London Electrician*, Vol. 31, etc.) Figures 102 to 104 show the arrangement plainly. The armature winding may be any of the usual types, but the ends of the coils are connected together without bringing them to the commutator. The commutator connections are made through what Mr. Sayers calls **Commutator Coils.** The commutator coils, which are shown by the heavy lines in the figures, each connect a commutator segment with one of the main coils which has an angular

position ahead of its commutator coil about equal to one-half of the distance between the tips of the pole pieces. There are thus as many commutator coils as commutator segments and main coils, but current is flowing only through those connected to the commutator segments which are at any instant in contact with the brushes. The relative angular positions of the commutator coils and their respective main coils place the main coil in the neutral position when its

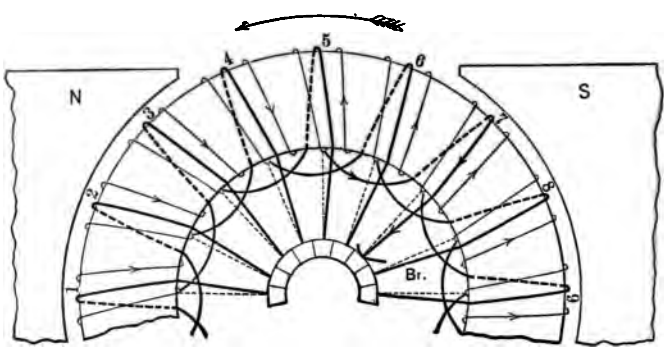


Fig. 103.

commutator coil is directly under the trailing or strong pole corner. The figures, 102 to 104, show by means of arrows the direction of the induced electric pressure in the coils, and the action of the commutator coils in controlling the current in the short-circuited coils is made plain. By controlling the number of turns in the commutator coils and varying their angular position with reference to the main coils, the commutation can doubtless be made practically sparkless without regard to the position in the field of the short-circuited coil.

Mr. Sayers has even gone farther than this in making a dynamo which operated satisfactorily with a considerable backward lead so that the armature magnetized the fields and no field winding was required. The operation of an ordinary machine with sufficient backward lead to magnetize the fields from the back turns of the armature causes vicious and destructive sparking and severe heating, because the short-circuited coil is placed in a strong direct field which tends to uphold

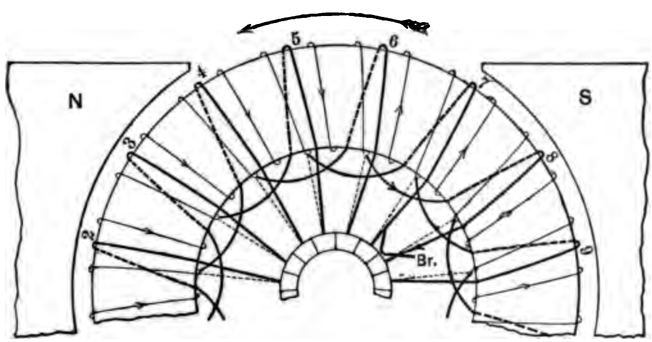


Fig. 104.

or even increase its current, while satisfactory operation requires that the current be reversed. It is evident that more turns are required in the commutator coils to effect their purpose when the machine is operated with a backward lead than when the commutating plane coincides with or leads the neutral plane.

A constant pressure machine may be caused to regulate with an astonishing precision by the use of the Sayers device. The accompanying table shows some observations taken by Mr. Sayers upon the operation of

a double horseshoe machine with embedded armature conductors, but without field windings. The backward lead of the brushes was kept constant.

SPEED.	PRESSURE.	CURRENT.	SPEED.	PRESSURE.	CURRENT.
918	101	0	916	100	58
920	100	0	940	100	57
928	101	34	966	101	62
942	103	35	964	100	80
906	100	58	964	99	78

Little practical experience has been had as yet with the Sayers device, but it seems to offer a satisfactory construction for some cases. In modified form it may possibly serve to reduce the evils due to armature cross-turns to a practical minimum if applied to the ordinary types of machines, thus making it easy to utilize fully the advantages of machines with embedded armature conductors and small air spaces, though its general adoption is doubtful.

While armature reactions is the fundamental cause of a variable lead, and its resulting sparking, yet the evil effect may be attacked with some success from an entirely external position. Thus any device which makes a considerable lead possible on light loads, enables a machine to operate with little or no change of lead under varying loads. Placing an extra resistance in the circuit of the short-circuited coil serves the purpose. There are several methods of doing this, the most convenient of which are: using a high resistance brush, such as carbon; and connecting the armature coils to the

commutator divisions by wires of considerable resistance, such as fine German silver wires. The carbon brush has come into very general use for machines designed to operate on a pressure of not less than 200 volts, and gives excellent satisfaction. It is not so satisfactory for lower pressures on account of the great cross-section of the brushes and size of the commutator required to prevent undue heating. Thus, satisfaction is given by copper brushes when carrying 100 to 250 and even up to 500 amperes per square inch of contact between brushes and commutator, while carbon brushes cannot carry satisfactorily much more than 50 to 60 amperes per square inch. The immediate effect of increasing the resistance in the circuit of the short-circuited coil is very simple, though it doubtless is accompanied by more complex phenomena. When working at full load the short-circuited coil must be in a field of considerable strength in order that its current be properly reversed. If the load falls off and the lead is not reduced, the field tends to cause an excessive flow of current through the short-circuited coil. The extra resistance checks the excessive current and thus reduces sparking and heating. When constant pressure dynamos are operated under fairly constant loads and receive careful attention, the high resistance brush or its equivalent is unnecessary; but where the loads vary rapidly or the machine is neglected, the high resistance brush is invaluable. This explains the popularity of the carbon brush for use on street railway generators and motors and for general power service. Evidently a comparatively large extra resistance for the electric pressure of the short-circuited

coil to overcome will cause comparatively little loss in the total pressure due to the main current passing through it. The currents are as 1 to 2 in ordinary constructions, while the ratio of the electric pressures is likely to be smaller than 1 to 50.

The thickness of the brush itself may have a marked influence on sparking. The effect in most cases is doubtless a complex one, in which the length of the interval of short circuit, and the self-induction, mutual induction, and resistance of the short-circuited coils all play a part. That self-induction and resistance play a considerable part is shown by dividing the brushes of a machine (Fig. 105), and placing the two portions of each brush several commutator segments apart. In this case commutation can be satisfactorily effected in a stronger field than when single brushes are used, and to gain sparkless commutation in a field of fixed strength the number of segments included between the brush parts must be decreased as the load decreases. The effect of self-induction is also shown to some degree by the propensity of ring armatures to spark with less apparent reason than drum armatures. An illustration of the effect of mutual induction has already been suggested in explaining the action of motor-generators (page 188). A complete study of the effect of self-induction or of the width of brush contact has never been made, but that sparking may be considerably affected by changing the thickness of the brushes is well known (*vide* Weymouth, "Drum Armatures and Commutators," Chap. 20).

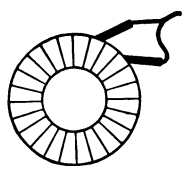


Fig. 105.

CHAPTER VII.

CHARACTERISTIC CURVES, AND REGULATION.

FOUR curves which represent relations between E, N, $\overline{nc}$, etc., are usually called characteristics, a name first applied to one of them by Deprez. They are:

1. The *Curve of Magnetization*, or *Hopkinson's curve.*
2. The *External characteristic.*
3. The *Loss line.*
4. The *Magnetic distribution curve.*

The **Curve of Magnetization** is essential to a proper study of the economical performance of a dynamo, and it is a valuable check upon the calculation of the magnetic circuit. The practical determination of the curve is simple, and it should be carried out by builders, for each new size or type of dynamo constructed. The dynamo to be examined is run at a convenient constant speed, and the field is magnetized from an external source of current, which is variable at will. The magnetizing current is measured by an amperemeter, the pressure at the dynamo brushes by a voltmeter, and the speed is taken by a convenient counter. The readings of the voltmeter should be corrected to a constant speed, if this varies, and the amperemeter and

corrected voltmeter readings should be plotted on rectangular axes with such a scale that $\overline{nc}$ and N_a can be directly read off. The curve of magnetization of a dynamo should not be regarded merely as a curve representing the relation between amperes in the field and volts at the brushes. The curve is of much value in determining, by comparison, the *relative* performance of dynamos, and for this purpose it must be plotted with such scales that it represents at once the relation between $\overline{nc}$ and N_a. It is usually plotted with $\overline{nc}$ on the X, or horizontal, axis, and N_a on the Y, or vertical, axis. The ratio of the scales upon which $\overline{nc}$ and N_a are plotted makes a decided difference in the appearance of the curve, and a uniform ratio should be adopted. When 100 magnetic lines of force are plotted to the same length as one ampere turn, the curve usually takes a convenient form, but this depends somewhat on the type of the machine to which the curve belongs. By applying a proper series of scales to a curve of magnetization, the same curve can evidently be used to represent the pressure at the brushes, as a function of $\overline{nc}$; or, when a constant pressure is maintained, it may represent the speed, as well as the total magnetization passing through the armature, as a function of $\overline{nc}$. The effect of hysteresis is plainly marked by a difference in the curves of magnetization, which are determined with increasing and decreasing currents. In the actual working of dynamos, the magnetization for any load falls in the descending or ascending branch of the curve, depending upon whether the load has previously been greater or less. In any case, the differences in

magnetization due to this cause are not great, and the average of the branches is taken as practically representing the working curve of magnetization.

The curve of magnetization starts from the Y axis, at a distance above the origin which is proportional to the residual magnetism. Since, when B is small the value of μ increases rapidly as B increases, the first part of the curve is convex to the X axis. After the first curvature, the curve of magnetization tends to the direction of a straight line, which it follows until B has become some thousands of lines per square centimeter, when μ begins to decrease rapidly, as B increases farther. The curve then deviates from the direction of the straight line, making a more or less abrupt bend, which is concave to the X axis. A little distance beyond the bend, or knee, the curve again approximates to a straight line, this time more nearly parallel to the X axis. The value of $\overline{nc}$ at each point of the curve can be divided into three portions: first, the number of ampere turns required to force the lines of force through the air space; second, the number required to force the lines through the field magnets; third, the number required to force the lines through the armature core. These component values can be plotted with N_a, giving partial curves of magnetization for the air space, magnet frame, and armature core. For any value of the ordinates, the sum of the abscissas of the partial curves is equal to the abscissa of the curve of magnetization. The partial curve representing the air space is a straight line passing through the origin, since the permeability of air is constant and the retentiveness is zero. It is

this line which the total curve approximately follows in the lower part of its course (between the two bends). The partial curves for the field magnets and the arma-

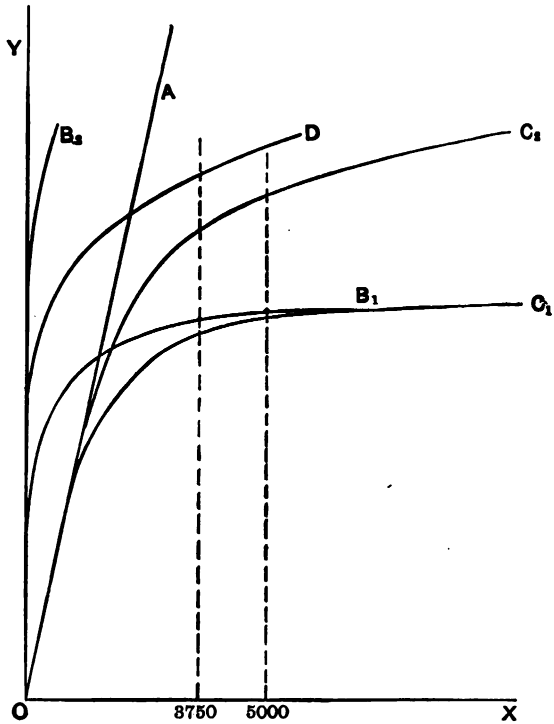


Fig. 106.*

ture are evidently curves of magnetization, representing the quality of the iron used in these parts of the dynamo (see page 35). Figure 106 shows the curves of

* Adapted from *Transactions American Institute of Electrical Engineers*, Vol. 4, p. 148, "The relation between the cross-section of the iron in armature and field of the Gramme dynamo."

magnetization of a dynamo where two different ring armatures were used with the same field magnets. The armatures were made with different radial depths, so that one had 2.4 times as great a cross-section as the other. Their external diameter was the same, so that the air space was constant. The line OA, in the figure, is the partial curve for the air space, OD represents the magnet frame, OB_1 and OB_2 represent respectively the small and large armature cores. Curves OC_1 and OC_2 are the total curves taken with the different armatures. The lower parts of both the total curves follow the air space line very closely, while OC_1 deviates from the line much earlier than OC_2. That this is due to the saturation of the armature core is made evident by curve OB_1. The curves of magnetization show the output of the large armature to be 26 per cent greater than that of the small one, when the ampere turns on the field number 3750, while the difference in the weight of the machine, due to difference in the armatures, is only about 7½ per cent. The difference in weight of armature copper is inappreciable. For more stable working the machine might be magnetized by 5000 ampere turns, in which case the large armature gives 33 per cent greater output than the small one. The increase of the magnetizing force from 3750 to 5000 ampere turns, increases the output of the small armature only 3 per cent, while the output of the large armature is increased 9 per cent.

In the original design of the machine these results should be obtained with a fair degree of accuracy by calculation, but the curves which are taken after the

machine is built evidently give the designer an excellent check upon his work. The partial curves can be made useful in determining whether excessive saturation exists in any part of the magnetic circuit, which may have been overlooked in making the design. The small armature in the example given was saturated at 3750 ampere turns, to a value of $B = 17{,}500$. B in the large armature, for the same number of ampere turns, was about 10,000. In the field cores, B was respectively about 12,500 and 16,000. Other things being equal, no other comment than these curves is required to establish the commercial economy of the larger armature.

The **External Characteristic** has different forms for series, shunt, and compound wound dynamos. To experimentally determine the external characteristic of any dynamo, it is made to excite itself, while the volts at its terminals, and its current, are measured with different resistances in the external circuit. The observations may be plotted in a curve using volts as ordinates and amperes as abscissas. In a series dynamo, the total current from the armature magnetizes the fields, the volts increase with the current, and the curve is very similar to the curve of magnetization. It always falls below the curve of magnetization, however, the difference of the ordinates being equal to the loss of pressure due to the armature reactions and the resistance of the windings of the machine. With a constant lead, according to the previous discussion of armature reactions, the difference in the ordinates of the two curves is practically proportional to the current, and can be

represented by a straight line. This line is the third form of characteristic curves, or the **Loss Line.** When the lead varies with the current, the back turns vary as a function of both the lead and the current, and the loss line may become considerably convex toward the *X* axis.

Figures 107 and 108 show characteristic curves experimentally determined from dynamos while the lead was kept constant. In Fig. 107, *A* and *B* are respectively

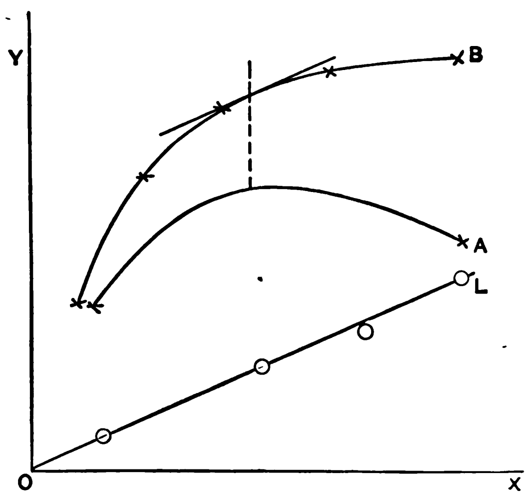


Fig. 107.

the external characteristic and curve of magnetization of a series dynamo. *OL* is the loss line, which represents the mean direction of the points marked ⊙. These points have ordinates equal to the difference between the ordinates of the two curves, and their positions show no tendency to uniform curvature con-

vex toward the X axis, which is according to the theory of armature reactions already developed. In Fig. 108, A and B are the external characteristic and curve of magnetization of a shunt dynamo without lead. The points marked ⊙ are points on the loss line determined by a construction which is given later. By the theory of armature reactions, the lead being constant, these points should be located upon the line of loss due to

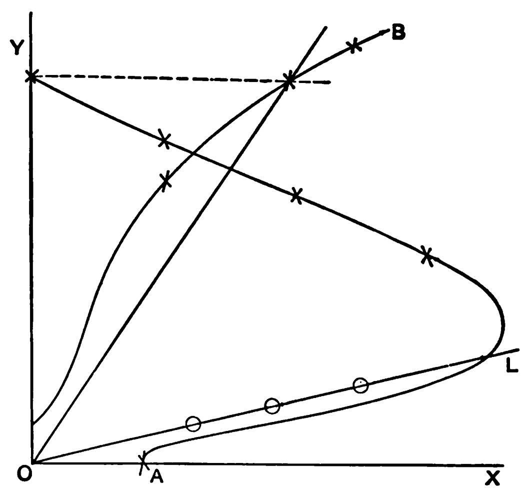


Fig. 108.

armature resistance. OL is such a line and the points are located upon it within the limits of error in the determination of their position.

The curve A in Fig. 107 shows that for a certain value of the current, a series machine will give a maximum pressure at the poles. For larger or smaller currents the pressure at the poles will be less. The

pressure is evidently at its maximum when the loss line is parallel to the curve of magnetization, since the total pressure increases faster than the internal losses up to that point, and beyond that point the losses increase more rapidly than the total pressure.

As already said, the loss line of a shunt dynamo can also be derived from the curve of magnetization and the external characteristic, and if the curve of magnetization and loss line be known, the characteristic can be directly derived; but, as shown in Fig. 108, its form is very

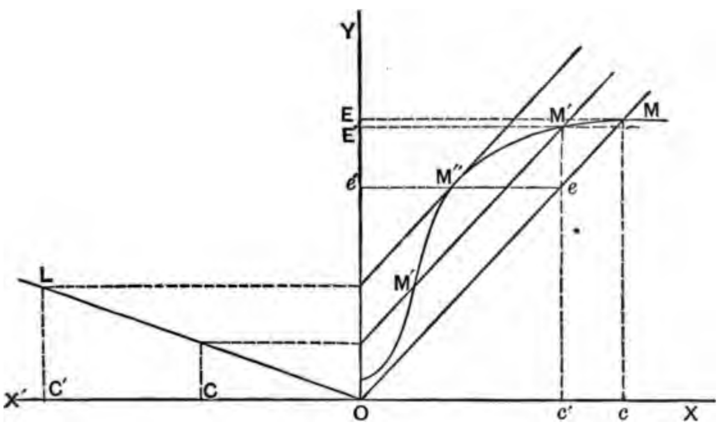


Fig. 109.

different from that of the external characteristic of a series dynamo. Suppose that the curve M in Fig. 109 represents the curve of magnetization of a machine and OL the loss line. For convenience the curves are plotted on opposite sides of the Y axis, but the abscissas of both are measured from O. With a current $c = Oc$ in the shunt field coils, the pressure $E (= OE)$ is devel-

oped by the armature, giving the point M on the curve of magnetization. If the machine excites itself with an open external circuit, the resistance of the field coils must be $R = \frac{E}{c}$. $\frac{E}{c}$ is equal to the tangent of the angle cOM, which is the angle that the line OM makes with the X axis. The line OM is therefore called the **Field Resistance Line.** Under the condition of self-excitation on open external circuit, the resistance of the field circuit must have a different value for each point on the curve of magnetization, and the field resistance line is always the line drawn between the origin and the point on the curve.

If the external circuit be closed the pressure at the brushes is reduced by armature losses; that is $e = E - \epsilon$, where e is the pressure at the brushes, E is the total pressure, and ϵ the loss of pressure due to armature resistance and reactions. With self-excitation, and field coils of fixed resistance, the current in the fields is at once reduced from $c = \frac{E}{R}$ to $c = \frac{e}{R}$. This further reduces the pressure at the brushes, and again reduces c and E, if the current C in the external circuit be kept constant. Figure 109 shows plainly that the decrease of the field current c, and therefore of the total pressure E, will continue until the difference in value of the corresponding ordinates of the curve of magnetization and the field resistance line equals the ordinate of the loss line for the external current C. Therefore, if the curve of magnetization, field resistance line, and loss line are plotted, the pressure at the terminals for any armature current may be obtained by construction, thus:

from the point on the loss line corresponding to current C draw a line parallel to the X axis. From the point at which this cuts the Y axis, draw a line parallel to the field resistance line. The ordinate of the point M', where this parallel intersects the curve of magnetization, is equal to E', and the corresponding ordinate of the field resistance line is equal to e, since $M'e=\epsilon$ by construction and $E'-\epsilon=e$. For most values of C the parallel cuts the curve twice, and there are two corresponding values of the pressure. For a certain value of the armature current the parallel to the field resistance line is tangent to the curve of magnetization and E and e each have only one value. For greater values of the armature current the parallel to the field resistance line does not cut the curve of magnetization at all, and the range of the dynamo is exceeded, unless the field resistance can be reduced. It is thus seen that with a constant field resistance the external current of a shunt dynamo has a maximum value at a certain pressure, and that for pressures, either greater or less, the external current must be smaller (compare with series machine). Hence, as the resistance of the external circuit is decreased, the current increases up to the maximum point and then decreases. When the resistance of the external circuit is reduced to zero ("short circuit") the external current has a small value which is due to the pressure developed by residual magnetism.

If the loss line OL (Fig. 110), the curve of magnetization M, and the field resistance line OM are plotted, the external characteristic may be constructed as follows: From any point a, on the curve M, draw the horizon-

tal line *ab* to the point where it cuts the *Y* axis. Draw the vertical line *ac* to the point where it cuts *OM*. Draw the horizontal line *cd* indefinitely. From *b* draw a line parallel to *OL* until it cuts *cd*. This intersection is the point in the external characteristic which is sought. For, by construction, $C'd = ca' = e$, and $OC' = OC$, which is the armature current which causes a loss of pressure in the armature equal to *ac*. The external character-

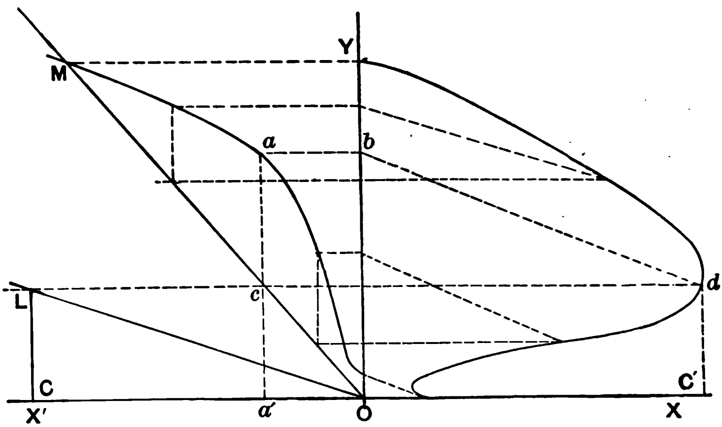


Fig. 110.

istic has a hook on its lower end, due to the portion of the curve of magnetization which is convex to the *X* axis.

If it be desired to make a shunt dynamo as nearly as possible self-regulating (*i.e.* to maintain a constant pressure under varying loads), it is shown by the construction that the loss line must have the smallest possible inclination to the axis of *X*, the field resistance must be small, and the field magnets well saturated.

The upper limb of the external characteristic is then as nearly horizontal as is possible. The first of these conditions demands that the armature resistance and the lead be the smallest possible, which requires that the armature be fairly large, and well balanced electrically and magnetically. The other conditions require considerable copper in the field coils, and a large C^2R loss in them. Self-regulation is usually obtained by compounding, that is by the addition of some series coils on the field magnets. By this means an external characteristic which is almost flat, over a considerable range of load, can be obtained. This will be discussed subsequently (page 216). In an experimental determination of the external characteristic, the speed should be normal and constant, as speed corrections cannot be accurately applied.

Various empirical formulas have been suggested to represent the general form of curves of magnetization, and the relations between armature pressure and current. One of the most thoroughly developed was first suggested by Frölich about 1881. Frölich's formula was based on an earlier formula suggested by Lamont. It may be taken as the type of all empirical formulas relating to the operation of dynamos, and is therefore given some attention here, although the determination of the value of constants that are required, in order that the formula may be applied to any particular case, is practically equivalent to the experimental determination of the characteristics.

The fundamental assumption of Frölich's formula is that the susceptibility of iron at any given intensity of

magnetization is proportional to the difference between that intensity and the maximum value which the intensity of magnetization can reach.

That is, $$\kappa=\frac{I}{H}=g(I_m-I),$$

and hence $$I=\frac{gHI_m}{1+gH}.$$

Since $B=H+4\pi I$, and $H=\frac{4\pi\overline{nc}}{10l}$, the formula may be approximately written,

$$B=\frac{a'H}{1+b'H}=\frac{a\overline{nc}}{1+b\overline{nc}}.$$

This is the formula of a hyperbola which passes through the origin and is asymptotic to a line which is parallel to the X axis. That it fairly represents the true form of the curve of magnetization within narrow limits is shown by the reciprocal or reluctivity curve (Fig. 26), which is a straight line (*vide La Lumière Électrique*, Vol. 39, page 492). In the practical designing of dynamos the formula is useless, since the numerical values of the constants a and b can only be derived by extensive experiments on each machine, and the effects of leakage, armature reactions, etc., are not included.

The *fourth curve* is useful in studying the distribution of magnetic lines over the pole faces, and the effect on the distribution caused by varying the cross-turns and self-induction of the armature. The curve may be plotted either on rectangular co-ordinates, or on what may be called polar co-ordinates, using for the origin the circumference of a circle. The

machine to be examined must be run at a constant speed and carrying the load for which it is desired to experiment. The magnetic distribution can then be determined by either of the following methods :

1. *By pilot brush and voltmeter* (Fig. 111). A and B are the main brushes of the dynamo, P is a pilot brush, and V a voltmeter. When P is on the same commutator bar as is A, the voltmeter will show no deflection. If P be moved by equal intervals toward B, the voltmeter will show increasing deflections until the pilot brush reaches B, when the deflection of V will be due to the pressure between the brushes A and B. The increments of the deflection are proportional to the number of lines of force entering, or leaving, the armature between corresponding positions of the pilot brush.

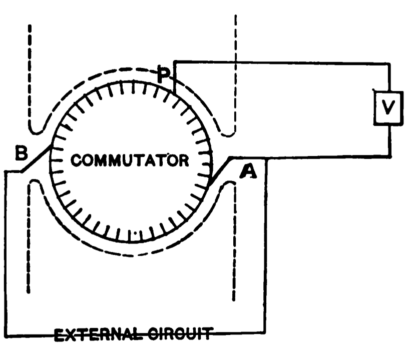


Fig. 111.

2. *By pilot brush and potentiometer* (Fig. 112). G is a galvanometer; C, a sliding contact on the resistance MN; MN, a slide wire, or similar resistance, of sufficiently large magnitude that only a small current will flow through it. When the galvanometer circuit is closed, and no deflection occurs, P and C are at the same pressure. The increments of pressure can thus be measured in increments of resistance, and the following proportionality is evident, $p = r\frac{P}{R}$; when P is the

total pressure in volts between A and B, R the resistance in ohms of MN, and p and r are their respective increments. This method is specially useful in making a careful investigation when the speed is variable, since

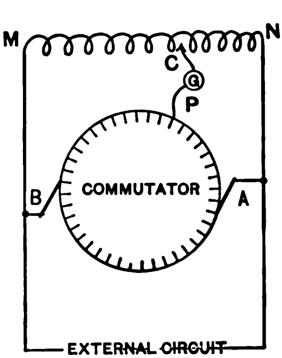


Fig. 112.

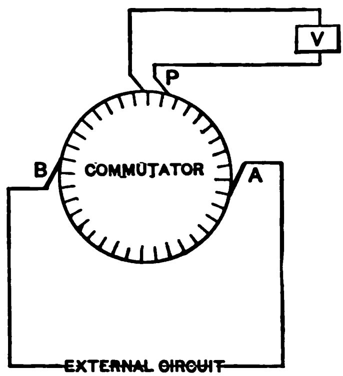


Fig. 113.

a considerable change in the speed will not appreciably change the *relative* distribution of the lines over the pole face.

3. *By a pair of pilot brushes* (Fig. 113). P is a pair of pilot brushes fixed together so that their toes are a conveniently small distance apart (usually the width of one commutator division). These are connected to a sensitive voltmeter, the readings of which give directly the increments of lines of force entering the armature. The pair can be moved from brush A to brush B, taking readings at convenient intervals, and, in this way, a direct determination of the distribution of the lines of force over the pole face may be made.

These methods are quite convenient; but their records,

for the regions near the main brushes, must be more or less in error, on account of self-induction in the short circuited coils, which masks the effect of the lines of force entering the armature. The following method is not as convenient, but seems to be free from most of the errors due to self-induction in the short circuited coils.

4. *By test coil on armature.* A fine wire coil of a few turns is wound over, or between, the coils on the armature. One end of the coil is connected to the armature shaft, and the other end to an insulated pin

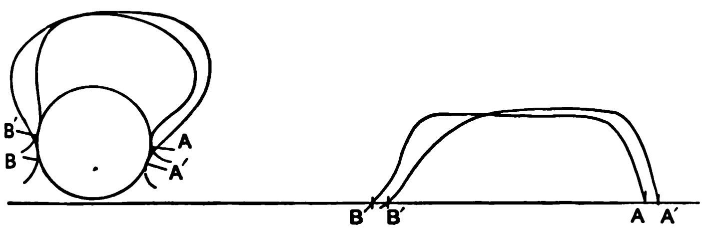


Fig. 114.

Normal curves of distribution of well-designed shunt dynamo. *AB*, open external circuit; *A′B′*, full load. Ordinates are increments of pressure, as measured by one of the preceding methods.

on the shaft. Between the brush on the shaft and a pilot brush, which touches the pin once per revolution, a condenser can be connected and charged. The charge of the condenser is evidently proportional to the instantaneous pressure in the test coil, at the instant of contact, and is therefore proportional to the rate of cutting lines of force by the test coil at that instant. The charge of the condenser can be measured by its discharge through a ballistic galvanometer. By moving

the pilot brush, the instantaneous pressure can be measured at all points of the circumference. An electro-

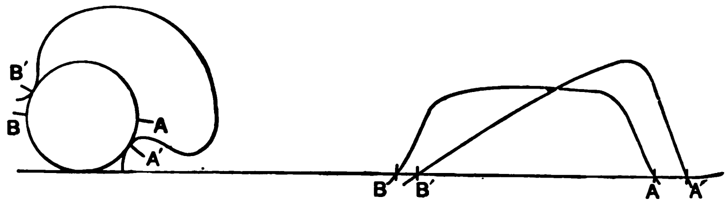


Fig. 115.

Curve of poorly designed shunt dynamo. Lead variable.

meter may be used in place of the condenser and galvanometer, if more convenient.

REGULATION.

Regulation for Constant Pressure. — *A. Shunt Dynamo.* A shunt dynamo with well-saturated fields and a loss line with small slope, as already shown (page 206), generates a fairly constant pressure, when driven at a constant speed, and under all loads not exceeding the maximum for which it was designed. The same machine when used as a motor fed at constant pressure, will run at a fairly constant speed under varying loads.

When the field is worked at its most economical saturation, it is too near the bend of the curve of magnetization to regulate well. Moreover, a constant pressure is not usually required at the terminals of the dynamo, but at some more or less distant centre of distribution. In this case, there is always a loss of pressure in the lead wires, which must be compensated

by a *rise* in pressure at the dynamo. When the dynamo is operated at constant speed, the pressure is a function of the ampere turns on the field magnets ($\overline{nc}$), and its regulation can be accomplished by three distinct methods used separately or in combination. These methods are:

1. *Variation of the current in the field windings.* In most cases where a shunt dynamo is used, the field winding is of less resistance than is required for proper magnetization under all usual demands (see p. 146), and a variable rheostat or "hand regulator" is included in the field circuit. By this means the field resistance, and hence the field current, can be varied through a considerable range, with a resulting variation in the pressure developed by the armature. The change of resistance in the field circuit which is required to produce any desired change in the pressure at the armature terminals, can be readily determined from the characteristic curves.

In the accompanying figure (116), OM is the curve of magnetization, OR, the line representing the resistance of the field circuit when the dynamo is producing its normal pressure, OE, with the external circuit open, and OL is the loss line. It is desired to know the reduction in the resistance of the field circuit which is necessary in order that the pressure at the terminals may remain OE, when current C flows in the armature. By construction, CL is the loss of pressure in the armature. Lay off $OE' = OE + CL$, or, what is equivalent, lay off $L'E' = OE$. Draw the horizontal $E'm$ and the vertical

mc'. The point where the latter cuts the horizontal Ee is evidently a point on the new resistance line. For, with current Oc' in the field windings, the total armature pressure OE' is produced, since m is on the curve of magnetization, but the pressure at the field terminal is equal to $c'e = OE$, hence the resistance of the field circuit must be equal to $\frac{OE}{Oc'} = \tan XOe$. Drawing a line from O through e gives the line desired, or OR'. The

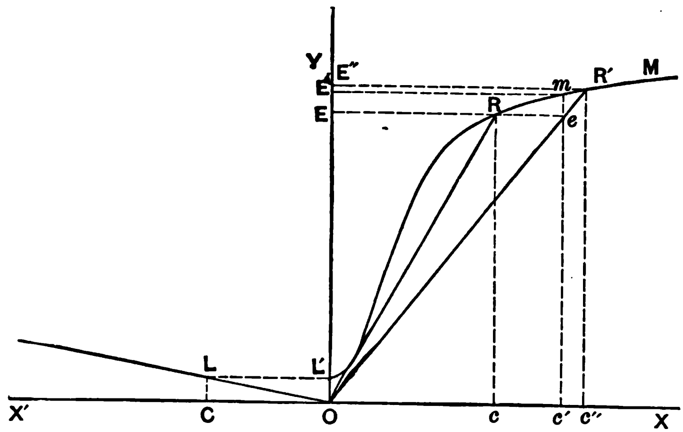


Fig. 116.

difference in the tangents to the angles XOR and XOR' gives at once the amount of change in the field resistances required to produce the desired result. With the external circuit open and the new field resistance OR', the pressure at the dynamo terminals becomes OE''. This is greater than the pressure OE', because the armature losses are negligible with open external circuit, and the pressure between the terminals of the field

windings is the total pressure developed by the armature. The magnetizing current in the field windings, therefore, is greater when the external circuit is open than when it is closed, if the resistance of the field circuit is not changed.

2. *Variation of the effective turns on the field.* A certain number of turns can be so arranged that the current through them can be reversed at will. By this means the effect of a portion of the turns can be neutralized without change of resistance. Such an arrangement gives a method of regulating, but it is expensive and unsatisfactory.

3. *Variation of both field current and effective turns.* A certain number of turns can be arranged so as to be short-circuited at pleasure, but it is evident that the change of magnetization due to the changing of turns is largely balanced by the attendant change of resistance and therefore of the field current.

The first method is the only thoroughly practical method of external regulation for a shunt dynamo. By it the field current can be varied through a considerable range and a considerable loss of pressure in feeders and mains can be duly compensated by regulating the pressure of the dynamo, so as to keep the pressure constant at distributing points. For this reason, the shunt dynamo with "hand regulator" is ordinarily used in central stations for incandescent electric lighting, as it is here necessary to keep the pressure practically constant at the lamps. The variations in the rheostat have sometimes been made automatically through mechanism actuated by a relay, which in turn depended for

its working upon the electric pressure at distributing points. Such arrangements are useless, however, where a number of dynamos are operated in parallel (see page 234), and compound wound dynamos are more satisfactory and have supplanted the "automatic regulator."

For isolated or private incandescent lighting plants, the loss in the wires is usually comparatively small, and compound wound dynamos can be used to advantage. They are also used as generators in special power plants, where it is desirable to take care of rapid changes of load, which often cannot be properly provided for by hand regulation.

Compound wound dynamos are shunt wound machines with a certain number of series turns on the field magnets. The series turns are usually designed to be sufficient to make the external characteristic of the machine a nearly straight horizontal, or slightly rising, line.

They are of two classes:

1. Those with a short shunt, in which the terminals of the shunt field go to the brushes.

2. Those with a long shunt, in which the terminals of the shunt field go to the terminals of the machine (see page 137).

The latter class seems to give the best results, apparently, because the magnetization of the shunt coil remains more nearly constant and the regulation is effected directly by the series coils.

A method of predetermining the number of series turns required to give constant pressure has already been given (see page 158). The following methods are also interesting and useful:

Graphical method (Fig. 117). Having the curve of magnetization, external characteristic, loss line and shunt field resistance, it is desired to determine the number of series turns required to give a constant pressure for all loads.

Let the point P, on the characteristic, represent the current C, which is about the average load of the machine. The loss of pressure in the dynamo, due to this load, is $PP' = NQ$, and Oc' represents the ampere turns in the

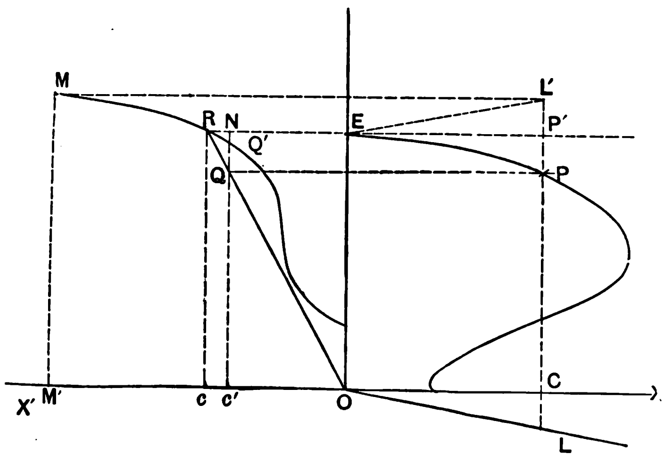


Fig. 117.

shunt field windings for this load. Of the total drop, NQ, in the pressure, the part QQ' is due to the loss of pressure in the armature, and the part NQ' is due to the decrease of current in the shunt field, the resistance of which is kept constant. With current C in the armature, in order that the pressure at the dynamo terminals may

be equal to $OE = CP'$, the total pressure must be $P'L = CL'$, where OL is the loss line. The field magnets must therefore be worked at M on the curve of magnetization. OM' is proportional to the total field ampere turns for armature current C. For a long shunt machine, Oc is proportional to the shunt field ampere turns, *i.e.* the number of ampere turns required to give the normal pressure OE, with open external circuit. Hence the number of series turns $= \frac{cM'}{C}$. The ordinates of the loss line should approximately include the loss of pressure due to the series turns, which can be readily estimated with sufficient accuracy. Unless the curve of magnetization is a straight line between the points R and M, the pressure at the terminals will not be exactly the same at all loads. To determine approximately how much variation in pressure, due to the curvature of the curve of magnetization, must be expected, the construction may be reversed, starting at a point which represents the magnetization at the full load of the dynamo. For a short shunt machine, the construction is similar but the number of the series ampere turns is slightly less, since the pressure at the terminals of the shunt field, and hence the shunt field current, increases with the load. If it be desired to keep the pressure constant at the terminals of a feeder, the pressure must rise at the dynamo with increase of load, *i.e.* the dynamo must be **Over-compounded.**

To find the series turns required to give any percentage rise of pressure, or over-compounding, the graphical construction is similar to that just given, but the loss of

pressure in the leads must be added to the ordinates of the loss line.

Analytical method. Let n and n' represent the number of shunt and series field turns respectively; B and X, the back and cross turns on the armature; R_a, the armature resistance; C and c, the currents in the armature and in the shunt field (C is also taken as the current in series field); E and e, the total pressure and the pressure at the terminals; P, the reluctance of the magnetic circuit of the dynamo, when the armature is idle $\left(=\Sigma\frac{l}{A\mu}\right)$; $P\frac{\sqrt{\overline{XC}^2+\overline{nc}^2}}{nc}$, the reluctance with current C in the armature; N_a, N_a', and N_a'', the lines of force passing through the armature under different conditions of working. Then $N_a=\frac{1.25}{Pv}nc$, when no current flows in the armature, and

$$N_a'=\frac{1.25}{vP\frac{\sqrt{XC^2+nc^2}}{nc}}\Sigma\overline{nc}=\frac{1.25a}{vP}[nc+n'C-BC],$$

when current C flows in the armature, ($\frac{1}{a}$ representing $\frac{\sqrt{\overline{XC}^2+\overline{nc}^2}}{nc}$, which is approximately equal to the increase in the reluctance of the path of the lines of force, due to the skewing effect of the cross-turns). The number of lines of force passing through the armature due to the shunt coils, is evidently $N_a''=\frac{1.25a}{vP}nc$, when current C flows in the armature. The magnetization, which is caused to pass through the armature by the shunt windings, is thus decreased by the distortion

of the field in the ratio of $a:1$. The series turns m make up this loss, and also compensate for the loss fr back turns and the electrical losses in the armatu The formula giving N_a'' is evidently correct only long shunt dynamos, in which c remains constant. T error for short shunt dynamos, however, due to consic ing c constant, is doubtless smaller than are the err in the assumptions from which the value of a is deriv

Now $E = \dfrac{V \times S \times N_a}{60 \times 10^8}$, and $E' = \dfrac{V \times S \times N_a'}{60 \times 10^8}$,

and $\qquad e = E' - CR_a$.

$$\therefore\ e = \frac{V \times S \times N_a'}{60 \times 10^8} - CR_a$$

$$= \frac{1.25 \times a \times V \times S}{v \times P \times 60 \times 10^8}[nc + n'C - BC] - CR_a$$

$$= \frac{1.25 \times a \times V \times S}{v \times P \times 60 \times 10^8} nc$$

$$+ \left[\frac{1.25 \times a \times V \times S}{v \times P \times 60 \times 10^8}(n' - B) - R_a\right] C.$$

The first term of the right-hand member may written

$$\frac{1.25 \times V \times S}{v \times P \times 60 \times 10^8} nc - \frac{1.25 \times V \times S}{v \times P \times 60 \times 10^8}(1 - a) nc$$

$$= E - \frac{1.25 \times V \times S}{v \times P \times 60 \times 10^8}(1 - a) nc.$$

$$\therefore\ e=E-\frac{1.25\times V\times S}{v\times P\times 60\times 10^8}(1-a)nc$$

$$+\left[\frac{1.25\times a\times V\times S}{v\times P\times 60\times 10^8}(n'-B)-R_a\right]C.$$

Since e at full load is to be equal to E, we must have

$$\left[\frac{1.25\times a\times V\times S}{v\times P\times 60\times 10^8}(n'-B)-R_a\right]C$$

$$-\frac{1.25\times V\times S}{v\times P\times 60\times 10^8}(1-a)nc=0.$$

Solving for n', we have

$$n'=\frac{1-a}{a}\times\frac{nc}{C}+B+\frac{v\times P\times 60\times 10^8}{1.25\times V\times S}\times\frac{R_a}{a};$$

but $\frac{v\times P\times 60\times 10^8}{1.25\times V\times S}=\frac{nc}{E}$, and $\frac{c}{C}\times 100$

= per cent loss in the field $=p$.

Therefore

$$n'=\frac{1-a}{100a}pn+B+\frac{ncR_a}{aE}.$$

The first term of the right-hand member of the equation gives the number of series turns required to compensate the effect of the cross-turns in skewing the lines of force. The third member gives the number of series turns required to compensate for the loss in armature resistance. To add the correction required to compensate for losses in the short-circuited coil at com-

mutation would require another factor in the third term, the application of which must be most uncertain. This formula probably can have no satisfactory application on account of the difficulty in finding a value of a in any particular case, which will be certain to fit, and on account of some uncertainty regarding the electrical losses in the armature. The value of a seems to vary in different types of machines from .75 to .85, causing the factor $\frac{1-a}{100a}$ of the first term average between .0035 and .002, and the factor $\frac{1}{a}$ of the third term 1.3 to 1.2. On account of the difference in the relative direction of the currents which flow in the field and the armature of a motor, as compared with that of a dynamo, the formula, when applied to a motor, becomes

$$n' = \frac{1-a}{100a}pn + B - \frac{ncR_a}{aE}.$$

In this case if the third term be greater than the sum of the first and second terms, n' becomes negative, and the winding must be **Differential** (*i.e.* opposed to the shunt winding) in order to effect regulation. If the third term be less than the sum of the first and the second, as is usually the case, n' is positive, and the series winding must be **Cumulative** (*i.e.* conforming to the direction of the shunt winding) to effect regulation. Finally, if the sum of the first and second term be equal to the third term, the motor will be self-regulating, without resort to series turns, for the weakening of the field magnetism, due to the effect of the cross and back turns, will be just sufficient to compensate the

effect of loss of pressure in the armature resistance. A comparison of the form of the formula, when applied to a dynamo or a motor, shows that the terminal pressure at a motor armature is greater than the total pressure generated in the same machine when the armature is driven as a dynamo at the same speed, and when the same current circulates in the field windings.

Since neither a, $\left(=\frac{nc}{\sqrt{\overline{XC}^2+\overline{nc}^2}}\right)$, nor B vary directly with C, the value of a differs slightly at different loads, and the regulation cannot be made perfect over the total range of load from zero to the maximum.

The armature speed does not enter directly into the final formula giving the number of the series turns, but any change in speed evidently changes E (the electric pressure with open external circuit), and c must vary proportionally. A change in the value of c changes the value of a slightly and of p proportionally; hence the first term of the formula varies directly with c. The third term varies inversely with c since $\frac{c}{E}$ is constant for all speeds, if the resistance of the field circuit is not altered. It is consequently evident that the number of series turns which effect practically perfect regulation at one speed, are insufficient at higher speeds, and cause over-compounding at lower speeds. This is plainly shown by the graphical construction, Fig. 117; for at increased speeds the point R on the curve of magnetization corresponding to pressure E, is higher up on the curve, since E is greater. Hence the reluctance of the magnetic circuit, when

magnetized by the shunt coils, is increased, and the effective magnetization due to n' series turns is decreased proportionally. For decreased speeds, the opposite effect takes place. Within the variation of speeds likely to occur in practice, the loss of regulation of a well-designed compound dynamo due to this cause is of small moment.

If the resistance of the field circuit be varied so that E remains constant regardless of changes in the speed, a somewhat different condition exists. The current c, in the shunt field windings, must be varied inversely with the speed instead of directly, as before, and both the first and third term of the formula vary directly with c, the first term varying more rapidly. This shows that the number of series ampere turns effecting regulations at one speed will not be correct at other speeds. The magnetization due to the shunt field is decreased so as to keep E constant at higher speeds. Hence the effect of the series turns is greater, and over-compounding results. In the same manner, under-compounding is the result, at speeds lower than the normal. This, again, is readily seen from the graphical construction by simply causing the scale of the ordinates of the curve of magnetization to vary with the speed. With a rise in speed the point R moves down the curve, and the reluctance met by the series windings is decreased. With a fall in speed the opposite effect occurs. In commercial service it is not always possible to operate a dynamo at the exact speed for which it was designed, and means for varying the pressure and effective compounding are desirable. For instance, standard dynamos used for

incandescent lighting may differ in speed as much as 10 to 15 per cent from that for which the machines were designed, and they may be expected to generate any pressure between 100 and 125 volts, with exact regulation or with over-compounding to suit local conditions. To obtain the desired flexibility, a variable rheostat, or hand regulator, is usually placed in the shunt field circuit (the windings being designed with the proper margin in resistance, see page 146), and a variable shunt is often placed across the terminals of the series coils.

Regulation for Constant Pressure. — *B. Series dynamo.* An inspection of the external characteristic of a series dynamo shows at once that it is impossible for it to produce a constant pressure when excited simply by the series coils.

Regulation for Constant Current. — *A. Series dynamo.* For common use in arc lighting, and for some other purposes, a current of constant strength called a **Constant Current** is desired. To produce a constant current, series wound dynamos are now almost invariably used. Series machines are also used as motors on constant current circuits, and sometimes on constant pressure circuits, when a readily variable speed and torque are required. The latter will not be discussed here, as such a discussion belongs essentially to the course in electric railways. In regulating a dynamo so as to produce a constant current, it is evident that the electric pressure at the terminals of the machine must be varied in exact proportion with the resistance of the external circuit, for $E = CR$, and C is required to be

constant. The external characteristic of a *self-regulating* constant current dynamo is a straight line parallel to the Y axis, and at a distance from it equal to the required constant current. It is therefore equivalent to the horizontal projection of a portion of the curve of magnetization upon a vertical plane passing through C.

In Fig. 118, OM, ON, OL, and AC represent respectively the curve of magnetization, common external characteristic, loss line, and self-regulating external characteristic of a series dynamo, arranged to so regulate as to produce a constant current. The figure makes it obvious that in order to obtain the greatest economy of construction, the field windings of a constant current series dynamo must contain such a number of turns that the current line AC passes through the highest point in the common external characteristic ON, when this is plotted with currents, or their equivalent, on the X axis; in other words, the machine should develop its maximum electric pressure when magnetized by the maximum number of ampere turns nC. In order that the maximum pressure may not be curtailed by the speed, if it is slightly less than the normal, it is well to add a few extra turns, say 10 per cent, to the number required for greatest economy of material. If the number of turns on the field be not sufficient to cause the maximum pressure shown by the curve ON in Fig. 118 to be developed, the total capacity of which the armature and frame are capable will not be utilized; while if the turns be in excess of the number required to develop this maximum, plus a proper margin, the additional copper is

wasted. When the total pressure developed by the machine is $AC = OE_m$, the maximum pressure available

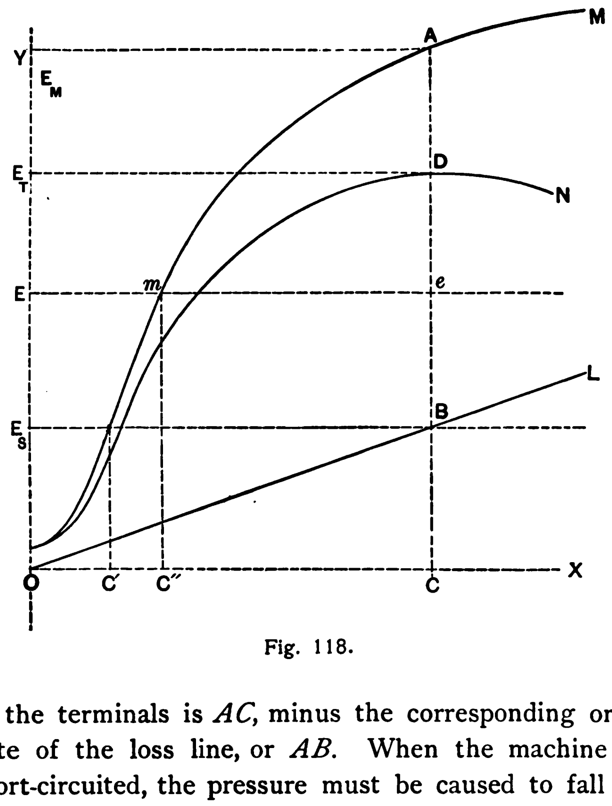


Fig. 118.

at the terminals is AC, minus the corresponding ordinate of the loss line, or AB. When the machine is short-circuited, the pressure must be caused to fall to the numerical value of the ordinate CB of the loss line ($= OE_s$), and the effective ampere turns must be reduced, from nC, which is proportional to OC, to a number proportional to OC'. It is easy to determine, by construction, the proper magnetizing power, when the external circuit has any resistance between zero and the maxi-

mum for which the machine was designed. Thus, if the resistance of the external circuit be R, the terminal pressure must be equal to CR, or e; from B lay off a distance Be numerically equal to e. Through the point e draw the horizontal line eE, from the point at which it cuts the curve of magnetization m, drop the vertical line mC'', and OC'' is proportional to the ampere turns required. In order that the strength of the fields of a machine regulated in this manner may never be reduced so far as to cause undue sparking, the loss line must have a large slope and the maximum magnetization must be high up on the curve of magnetization. In the case of arc light dynamos, another condition which requires a large slope in the loss line, and a high degree of saturation in the fields, is the sensitiveness of the arc lamps to slight changes of current, which influences the steadiness of the light. Any change in the slope of the loss line evidently changes the available maximum output of the dynamo, and also the ampere turns which are necessary to properly magnetize the machine at short circuit.

It is evident, from the preceding discussion, that the regulation of series dynamos to give a constant current can be effected only through external agencies; that is, compounding will not give the desired results, though it was early suggested, upon certain erroneous theoretical grounds, as a solution of the problem. Three methods present themselves, which may be used individually, or in combination, to effect the necessary changes of ampere turns. They are:

1. By shunting the field turns by a variable resistance.

2. By short-circuiting a variable portion of the field turns, or, what is equivalent, by cutting a variable portion of the field turns out of the circuit.

3. By varying the lead, and hence the effective turns on the armature and the back turns.

For any purpose which requires much more than ten amperes of current, the third method need not be considered, on account of the destructiveness of the sparking caused at the commutator, when the brushes are off of the proper commutating plane, and the current is of much magnitude. The first method is obviously the simplest, as its essential details consist of nothing but a variable rheostat connected across the terminals of the field winding. In order that the regulation may be automatic, the changes in the rheostat can be effected by the intervention of a relay which is in turn actuated by the line current. To use the second method, the field coils must be wound in sections, individual ends of which are brought to a conveniently placed series of consecutive terminal blocks. Through the intervention of a relay, or other actuating device, a slider may be caused to pass over the terminal blocks, short-circuiting or cutting out coils as required.

In arc lighting, for which purpose the greatest number of constant current dynamos are used, the third method presents various points of advantage, and the destructive effect of commutator sparking can be provided for, as the current seldom exceeds ten amperes. On account of the peculiar conditions to be met, every successful type of arc light machine must be the result of careful experimental development, and its discussion

does not come within the scope of this book, but belongs in a separate and later volume.

Regulation for Constant Current.—*B. Shunt dynamo.* Shunt wound dynamos have been frequently advocated for use as constant current machines on account of their sloping external characteristics, but they can be made to give a practically constant current only by externally varying the ampere turns on the field magnets, exactly as in the case of series dynamos. Shunt dynamos therefore present no marked advantages as constant current generators, when compared with series machines, and their field coils are more expensive to repair in case of damage. Again, the loss of energy in their field windings does not add to the useful slope of the loss line as in the case of series dynamos, and they consequently lack the stability which is necessary for arc light dynamos.

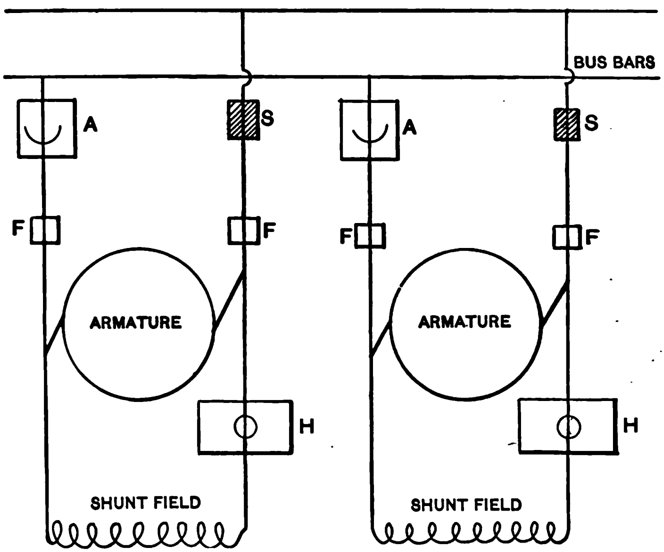
CONNECTING DYNAMOS FOR COMBINED OUTPUT.

Since it is neither possible nor desirable, in many cases, to furnish the whole output of an electric plant from a single dynamo, it is well to consider, from the strictly practical side, the methods of connecting two or more dynamos to one circuit in order that their outputs may be added. Also to consider the relation of such methods to the different modes of regulating. The connections may be arranged to put the dynamos in parallel (multiple arc), in which case the current capacities of the machine are added and their pressures must be equal; or they may be arranged to put the

dynamos in series, in which case the pressures are added and the normal currents of the dynamos must be equal.

Shunt Dynamos. — The simplest connection for dynamos is to put two or more plain shunt machines either in parallel or in series, for operation at constant pressure. This arrangement is used more frequently than any other for the purpose of furnishing a combined output, and is a very simple one. When the dynamos are connected in parallel, the terminals of each may be connected directly to the appropriate 'bus or main conductor through convenient indicating instruments, switches, and safety devices, and thus their like poles are connected together as shown in Fig. 119. Under these conditions each will carry its proper proportion of load without special precautions, if the machines are started with the same initial pressure and the speeds remain fairly constant. Any inequality in the loads which may occur through any cause may be readily corrected by means of the hand regulators, and the overpowering of one machine by another is rare, because a rise of pressure at the terminals of a shunt dynamo tends to increase the current developed by the armature, and an increased draught of current from the armature causes a decrease in the effective pressure, and hence shunt dynamos have a tendency to equalize their loads. Even if the pressure of one machine falls considerably below that of its mates, harm can only result from its conversion into a motor, and since its direction of rotation is the same either as a dynamo or as a motor, no greater damage than the blowing out of safety plugs is likely to occur.

When shunt machines are connected in series, the positive pole of one is connected to the negative pole of the next, and the outer poles are connected to the appropriate 'bus conductors. The fields may be excited



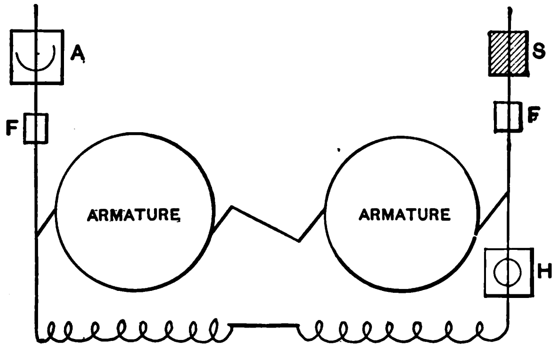
PLAIN SHUNT DYNAMOS IN PARALLEL.

Fig. 119.

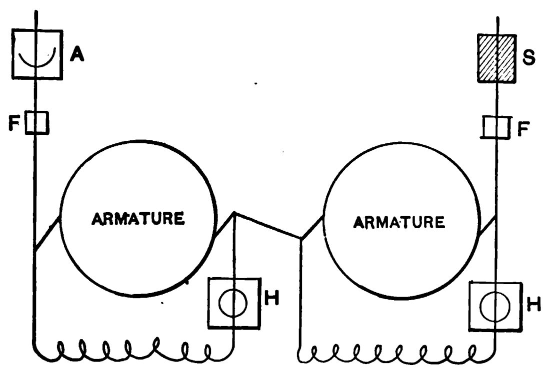
A, amperemeter; *S*, switch; *F*, fuse block or other safety device; *H*, hand regulator.

either by connecting them all in series with each other and with a single hand regulator, as in Fig. 120 *a*, or by connecting each field between the terminals of its own armature and in series with an individual hand regulator, as in Fig. 120 *b*. In the latter arrangement the

pressure generated by each dynamo may be adjusted as desired, but in the former arrangement regulation can



a. Fields in series.



b. Fields separate.

SHUNT DYNAMOS IN SERIES.

Fig. 120.

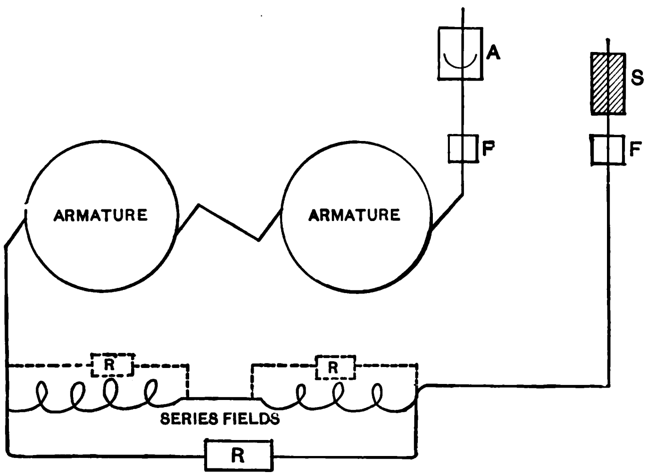
only be effected for all machines simultaneously and as a unit.

Attempts to automatically regulate two or more shunt dynamos by separate regulators so that they will give constant pressure when they are connected in parallel, usually result in failure, because any undue change of pressure at one dynamo affects the average pressure at the 'bus bars, and thus affects each automatic regulating device, and causes racing. Moreover, if the regulators are not equally sensitive, the dynamos will divide the loads in unequal proportions. The difficulty can sometimes be overcome by connecting the fields in parallel with each other and in series with a single regulator. Since compound dynamos are now used almost universally when automatic constant pressure regulation is desired, this method has practically dropped out of use.

Series Dynamos. — Plain series wound dynamos will operate without special precautions when connected in series. If it be desired to vary the pressure developed, a variable shunt may be connected around one or both of the field coils as in Fig. 121. Attempts to operate two or more series dynamos automatically regulated to produce a constant current, usually results in failure on account of racing, as in a similar manner automatically regulated constant pressure shunt machines fail when connected in parallel. The racing can be avoided by casting off the regulating device of all the dynamos except one, which throws the responsibility of regulation upon one machine.

An entirely different set of conditions exist when series dynamos are connected in parallel, as an examination of the external characteristics shows. If two or more series machines, connected in parallel, be started

with a proper division of the load, but the pressure generated by one is caused to decrease, the latter at once takes a proportionally smaller part of the total load, and the current in its field coils is decreased. This in turn causes a further fall in pressure, and again there occurs a decrease in the load carried, until finally the pressure falls so low that the machine is overpowered, its field

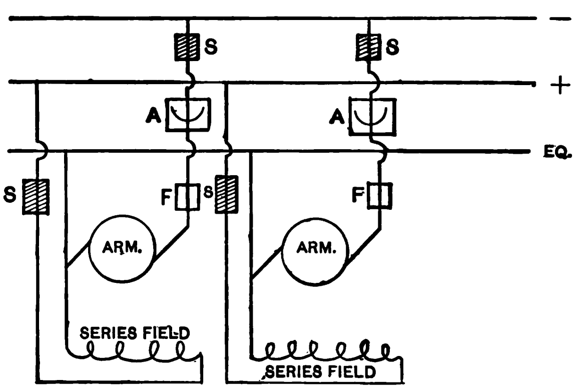


R, variable resistance.

Fig. 121.

is reversed, and it is converted into a motor. As a motor, its direction of rotation is evidently the opposite from its direction as a dynamo, and the result of reversal may be very serious. To prevent any machine being robbed by its mates, it is necessary to connect all the field windings in parallel with each other, as first suggested by Gramme. This can be effected by making a low resistance connection between the armature

ends of all the field coils, as shown in Fig. 122. Such a connection is called an **Equalizing Connection** or **Equalizer.** If the equalizing connection be of small resistance, when compared with the resistance of the series field coils, the distribution of the total current among the fields will evidently be independent of the output of any armature, exactly as would be the case if the fields were directly connected in parallel with each



Equalizing Connection.

SERIES DYNAMOS IN PARALLEL.

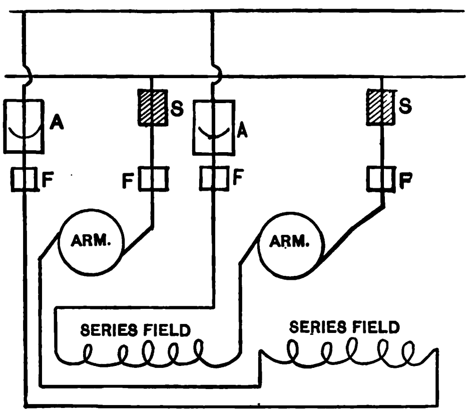
Fig. 122.

other. This arrangement evidently requires the field windings to be attached to brushes of the same polarity in all machines of a set.

When it is desired to operate two series dynamos of equal capacity in parallel, the equalizer may be dispensed with by connecting the dynamo fields, so that each is excited by the armature of the other machine, as in Fig. 123. This may be called **Mutual Excitation.**

It is not usual to run series dynamos in parallel, as they are seldom used to develop a constant pressure; but a discussion of the conditions of operation is useful from their analogy to the conditions in compound dynamos.

Compound Dynamos. Since compound dynamos may be treated as shunt machines with the addition of series



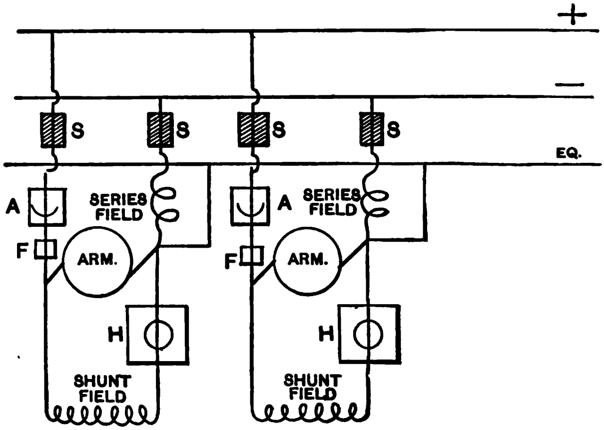
Mutual Excitation.

SERIES DYNAMOS IN PARALLEL.

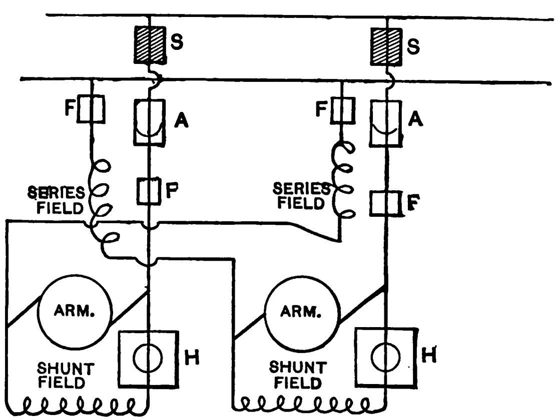
Fig. 123.

windings, analogy leads at once to the fulfilment of the requirements for connecting both shunt and series machines, when it is desired to run compound dynamos together. Consequently, when compound dynamos are operated in parallel, an equalizing connection is used to cause a proper division of the current amongst the series coils, as shown in Fig. 124*a*. This requirement was apparently first pointed out by Mordey.

When two compound dynamos of equal capacity are connected in parallel, "mutual excitation" of the series



a. Equalizing Connection.



b. Mutual Excitation.

COMPOUND DYNAMOS IN PARALLEL.

Fig. 124.

coils may replace the use of the equalizer, as shown in *Fig.* 124 *b*.

When the equalizing connection is used with compound dynamos, it evidently serves to equalize the magnetizing effect of the series windings, in the same way as it does in the case of plain series dynamos. It has already been shown (see page 200) that irregularities and differences in the curves of magnetization of different dynamos may cause considerable difference in their external characteristics, so that machines which run together perfectly at one load may not work together as well when carrying other loads, unless the magnetization of the field magnets be adjusted. In well-built, similar machines, the differences usually are small, and they may be readily compensated by means of the hand regulators. If for any cause the pressure of a compound dynamo falls much behind its mates, and compensation is not made, the current through its armature is likely to be reversed, and it is converted into a motor. The current through the series field windings is reversed at the same time, and it therefore opposes the effect of the shunt windings. If the series field is composed of many turns, the polarity of the magnets, and hence the direction of the armature rotation, may be reversed, with accompanying injurious results, exactly as in a series machine. If the field be not reversed it is materially weakened, so that a large current may flow through the armature for an instant, and damage it before the safety devices have time to operate. It is therefore of the utmost importance that the equalizing connections of compound dynamos be of ample cross-section, so that reversals will not occur, except by some very unusual accident. By a little care in adjusting the resistances of the series fields,

compound dynamos of different sizes, and even of different types, may be caused to operate in parallel satisfactorily.

In order that the division of the load between compound dynamos may be evident at all times to the dynamo tender, the dynamo amperemeters must be placed in the circuit so that all the current passing through the armature may be recorded, whether it flows through the main 'bus connections, or through the equalizing connections. This is effected by placing the amperemeter in the main 'bus connection which connects to the brush on the side opposite from the equalizing connection (see Fig. 124*a*). It is also usual to place a two-pole switch in the main 'bus connections, so that no current can pass through the armature by any path, when the machine is not in operation. For plain shunt machines, a single-pole switch in one 'bus connection is evidently sufficient.

Since both shunt and series dynamos can be operated in series without special precautions, analogy leads to the conclusion that the same is true of compound dynamos, and this proves to be true in practice.

When a dynamo is to be inserted, in series, into a circuit already operating, it should be brought to speed without any excitation, unless it be an automatic constant current dynamo, and should not be permitted to *build up* until it is properly connected in the circuit. The reason for this is that convenience requires the new machine to be short circuited while making the connections, if the working of the existing circuit must not be interrupted. When a dynamo is to be put into

parallel with dynamos already working, it must be brought to its normal speed, and to the pressure of the other machines, in order that it may not cause a disturbance among the other dynamos, or be itself overpowered. The simplest way of ascertaining when a dynamo, about to be placed in parallel with others, is at the proper pressure, is by comparing the brilliancy of incandescent electric lamps connected between its terminals, with other lamps connected to the circuit. Another method much used is to measure the pressure at the dynamo by means of a voltmeter; but the most satisfactory method, for general purposes, is the use of either a differential or a zero galvanometer as a *cutting-in galvanometer.* When the differential galvanometer is used, one coil is connected to the circuit and the other coil to the dynamo, so that their combined effect on a needle at their centre will be zero when the dynamo pressure equals the pressure of the circuit, and the needle deflection will therefore be zero. The zero galvanometer is composed of a single coil, with a needle at the centre, and consequently is less expensive and oftener used than the differential galvanometer. One terminal of the galvanometer coil is connected permanently to one 'bus conductor, and a terminal of the dynamo is connected to the other 'bus. If the free terminal of the galvanometer be connected to the free terminal of the dynamo, a current will flow through the galvanometer and deflect the needle, unless the circuit pressure and the dynamo pressure are exactly equal, and the direction of the deflection will show whether the dynamo pressure be too low or too high (Fig. 125).

The conditions for operating motors, when connected in parallel, or series, and geared or coupled to the same shaft, may be at once deduced from the preceding pages. Thus, shunt motors, connected in parallel upon constant pressure mains, will divide the load in proper proportion when they are geared to the same shaft, provided the ratios of the gearing are approximately equal. If the

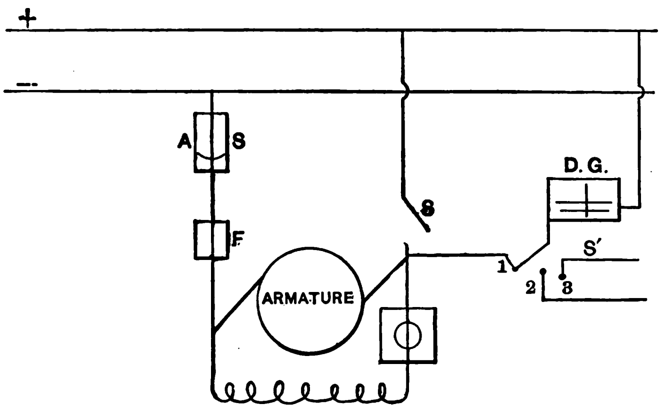


Fig. 125.

D. G., Dynamo galvanometer; *S′*, Small switch for connecting *D. G.* to any dynamo.

coupling of one motor is broken, it will not interfere with the operation of the others. Shunt motors, connected in series to main conductors, will operate satisfactorily when they are coupled to the same shaft, provided the coupling be positive. If the coupling of one motor slips or breaks, the motor will race, and rob the other machines of their proper proportion of the total pressure from the mains. Series or compound

motors, connected in parallel to constant pressure mains and coupled to the same shaft, are likely to divide the load badly, unless equalizing connections are used. Series or compound motors, connected in series to constant pressure mains, act in a manner similar to shunt motors. When connected to a constant current circuit, shunt and compound motors are entirely unsatisfactory, and series motors race badly under changes of load, unless some automatic device for regulating the strength of the fields is applied.

CHAPTER VIII.

EFFICIENCIES.

VARIOUS ratios have been denominated **Efficiencies** in the case of dynamos, but only three of them are of much importance. Each of the ratios has received several designations, but only the most applicable ones will be used here; these are:

1. *Efficiency of conversion;*
2. *Electrical efficiency;*
3. *Commercial efficiency.*

The definitions of the terms called efficiencies in the case of a dynamo are:

1. *Efficiency of conversion.* This is the ratio of the total electrical energy developed to the total mechanical power absorbed $\left(k=\frac{CE}{W}\right)$. It is always less than unity by an amount proportional to the loss due to friction, foucault currents, and hysteresis.

2. *Electrical efficiency.* This is the ratio of electrical energy in the external circuit (useful electrical energy) to the total electrical energy $\left[f=\frac{Ce}{CE}\right.$ for series dynamos and $f=\frac{Ce}{(C+c)E}$ for shunt or compound dynamos$\Big]$. It is always less than unity, by an amount proportional to the losses due to field and armature resistances.

a. *Series machines.* If f be the electrical efficiency, R, R_f, R_a the resistances of external circuit, field, and armature respectively, C the current, E the total pressure developed, and e the pressure at the terminals of the machine, then

$$f=\frac{Ce}{CE}=\frac{C^2R}{C^2R+C^2R_f+C^2R_a}=\frac{R}{R+R_f+R_a}=\frac{e}{E}.$$

From this it is evident that f is increased by decreasing the internal resistance.

b. *Shunt machines.* If c be the current in the field, and C the current in external circuit and armature, which may be taken as equal (this does not practically alter the value of the C^2R_a loss), then

$$f=\frac{Ce}{(CE+cE)}=\frac{C^2R}{C^2R+C^2R_a+c^2R_f}=\frac{\frac{e^2}{R}}{\frac{e^2}{R}+\frac{e^2R_a}{R^2}+\frac{e^2}{R_f}}$$

$$=\frac{\frac{1}{R}}{\frac{1}{R}+\frac{R_a}{R^2}+\frac{1}{R}}=\frac{1}{1+\frac{R_a}{R}+\frac{R}{R_f}}.$$

To find the relation between R, R_a, and R_f (R being the independent variable), at which f is a maximum, we have

$$\frac{df}{dR}=0=\frac{1}{R_f}-\frac{R_a}{R^2},\text{ whence } R=\sqrt{R_aR_f}$$

The electrical efficiency of a given shunt dynamo is therefore greatest (*i.e.* C^2R losses make up the smallest possible percentage of the output) when working under

a load at which the external resistance is the geometrical mean between the resistances of armature and field. In this case

$$f=\frac{1}{1+2\sqrt{\frac{R_a}{R_f}}}$$

Hence, if a machine is to be designed to give a fixed electrical efficiency, at a fixed load, the ratio $\frac{R_a}{R_f}$ has a perfectly definite value whatever the size of the machine. For instance, if $f=95$ per cent, $\frac{R_a}{R_f}=\frac{1}{1444}$, R_a and R_f being hot resistances. According to the tables given earlier, a 25 K.W. machine may have a loss of $2\frac{1}{2}$ per cent in armature and $2\frac{1}{2}$ per cent in the fields, which makes the electrical efficiency 95 per cent. The 25 K.W. machine of the example given earlier, has an electrical efficiency at full load of about 95.2 per cent ($R=.625$ ohms, $C=200$ amperes) and $\frac{R_a}{R_f}$ is about $\frac{.012}{22.5}=\frac{1}{1900}$. This machine would have its maximum electrical efficiency with a load of 240 amperes, when $R=.52$, and the electrical efficiency would be 95.7 per cent.

c. *Compound machines.* If R_f be the shunt field resistance, and R_s the series field resistance, then

$$f=\frac{C^2R}{C^2R+C^2R_a+C^2R_s+c^2R_f}$$

$$=\frac{\frac{1}{R}}{\frac{1}{R}+\frac{1}{R_f}+\frac{R_a}{R^2}+\frac{R_s}{R^2}}=\frac{1}{1+\frac{R}{R_f}+\frac{R_a}{R}+\frac{R_s}{R}}.$$

Solving for the relation between the resistances at which f is a maximum, we have

$$\frac{df}{dR}=0=\frac{R_a+R_s}{R^2}-\frac{1}{R_f},$$

whence $$R=\sqrt{(R_a+R_s)R_f}.$$

f is therefore a maximum, when the external resistance is a geometrical mean between the shunt field resistance and the sum of the armature and series field resistances. When this is the case,

$$f=\frac{1}{1+2\sqrt{\frac{R_a+R_s}{R_f}}}.$$

If x be the output of a machine and a the electrical losses, then

$$f=\frac{x-a}{x}, \text{ whence } (1-f)x=a,$$

This gives a relation between the output and electrical losses of a machine. The curve plotted for values of $1-f$ and x, taken from the tabulated losses given previously (pages 108 and 138), is asymptotic to a line representing an electrical efficiency of 97.3 per cent.

3. *Commercial efficiency.* This is the ratio of external electrical energy (output) to power absorbed $\left(\eta=\frac{Ce}{W}\right)$. If k be the efficiency of conversion, f the electrical efficiency, and η the commercial efficiency, then

$$\eta=k\times f.$$

a. *Series machines.*

$$k=\frac{CE}{W} \text{ and } f=\frac{e}{E}, \text{ whence } \eta=k\times f=\frac{Ce}{W}.$$

b. *Shunt* or *compound machines.*

$$k=\frac{(C+c)E}{W} \text{ and } f=\frac{Ce}{(C+c)E},$$

whence $$\eta=k\times f=\frac{Ce}{W}.$$

The commercial efficiency is an important measure of the duty of a dynamo, since, in making a comparison between two of equal cost and equal mechanical workmanship, the machine which gives the highest commercial efficiency is the most economical to the user. This is not only due to the power saved, but principally to the fact that in the dynamo of highest efficiency, there is less loss of power and consequently less heat developed in the windings and the armature core; the insulation, therefore, is less likely to depreciate, and repairs are therefore reduced. It is obvious that the preceding discussion of dynamo efficiencies applies equally to motors, but the equations become

$$k=\frac{W}{Ce},\ f=\frac{e}{E},\ \text{and}\ \eta=\frac{W}{CE},$$

in which e is the counter-pressure of the motor, and E is the pressure of the mains.

It is of considerable importance to analyze the causes of internal loss (heating), so that they may be attacked in detail in efforts to make the total loss a minimum. The principal causes of loss are:

1. *C^2R loss* in the conductors on armature and field.
2. *Foucault or eddy currents* in armature cores and pole pieces.

3. *Foucault or eddy currents* in armature conductors.
4. *Hysteresis* in armature cores.
5. *Friction* of bearings and brushes, and air friction.

The heating due to the C^2R loss can be directly calculated when designing a machine, as far as the loss in armature and field conductors is concerned. If the completed machine shows an appreciable loss due to commutator connections, brush contacts or brush connections, improved mechanical construction will suppress the evil.

The methods of reducing foucault currents in a core have already been discussed, but the following additional discussion is of importance. When the armature core is made of insulated iron discs, the loss by foucault currents in each disc $= \frac{e^2_d}{r_d}$, r_d being the electrical resistance in the path of the currents through the disc, and e_d being the pressure generated in the disc. e_d is proportional to the number of revolutions per minute made by the armature and to the number of lines of force entering the disc. The latter makes e_d proportional to to the thickness of the disc, while r_d is evidently inversely proportional to the thickness of the disc. Hence $\frac{e^2_d}{r_d}$ is proportional to the *cube of the thickness of the disc.* As the number of discs in an armature core is inversely proportional to the thickness of the discs, the total heating due to foucault currents in completely insulated discs is proportional to the *square of the thickness of the discs.* On account of mechanical considerations, it is not well to reduce the thickness of the discs to much

less than .015″ (15 mils). Hence, if two or more discs make electrical contact with each other, the heating may be considerable. It is therefore usual to put single sheets of tissue paper between the discs, although in some cases the black oxide covering of the discs is relied upon for insulation. After an armature core is assembled by the usual methods, its surface is not smooth, and it must be turned off. If a burr be formed across the discs in this operation, an excellent path for foucault currents is formed. To avoid this chance, it is usual to place a bunch of tissue sheets, or a disc of cardboard, at intervals of 5 or 10 iron discs. When the armature core is made of wires, as in some Gramme armatures, especially those of English manufacture, the contact of the wires is much smaller, and therefore there is less need for care in insulation. It is usual in this case to rely for insulation entirely upon the black oxide on the wires. The foucault current loss seems to vary between 20 and 60 per cent of the total loss in conversion. In well-built machines, of capacities not smaller than 5 K.W., it probably should not exceed 30 per cent of the loss in conversion.

Foucault currents in the armature conductors (often called parasitic currents) give little or no trouble, except in machines where the normal output requires large armature conductors. Beginning with machines for 500 amperes and upwards, the parasitic currents may become of marked magnitude, unless precautions are taken against their generation. In such machines, the armature conductors are of considerable cross-section, and can be made either of bars or of several wires

wound in parallel. When bars are used, they should be rectangular in section, and placed on edge upon the armature core. If placed on their sides, currents will circulate through the body of the bars on account of the difference in the rate of cutting lines of force, which may occur at the two edges. Where multiple wires are used, the coil should be given a half twist when crossing the head, so as to reverse the relative position of the individual wires in the coil, on opposite sides of the armature. This is necessary, in order that the wires may be of exactly the same length (and therefore of equal resistance), and so that they may cut exactly the same number of lines of force at any instant. With proper precautions, the loss due to foucault currents in armature conductors, can usually be reduced to a negligible quantity. In some machines of low voltage, and exceedingly large current output, such as are sometimes used in electro-metallurgy, there is more difficulty in avoiding waste from this source; but even in such machines the loss can be made quite small, by proper care in designing, and the difficulties involved are not as great as those connected with the collection of the large currents.

The loss by hysteresis, in the iron of small armature cores, should not be a large part of the total loss in conversion, but it is likely to be a very important element in large cores. In some cases, it may amount to 40 per cent of the total losses. The loss varies with B, and with the quality of the iron of which the core is composed. With a given specimen of iron and value of B, the loss per reversal, and per unit of iron, can be experi-

mentally determined. In the best soft iron used for armature discs, the loss is likely to be fully 0.00001 and not to exceed 0.00002 watts per cubic centimeter, and per complete cycle (revolution) per minute (see page 74). The loss due to hysteresis causes heating throughout the mass of the rotating armature core, and the heat is therefore not as readily dissipated as that due to the C^2R loss on the surface. Ventilation of the core assists in dissipating the heat; but with the best attainable ventilation, the temperature at the centre of the core increases rapidly with increasing diameters. With a constant surface velocity, and equal ventilation, the temperature at the centre of a Siemens core increases about proportionally to the square of the diameter. This condition is sufficient to limit the practicable diameter of Siemens armatures, for two pole dynamos, even if no other conditions entered.

The loss due to friction is likely to amount to from 15 per cent to 40 per cent of the loss in conversion, when the machine is driven by a belt. Its magnitude is largely a matter of mechanical workmanship, lubrication, and flexibility of the driving belt. It may be materially influenced by the magnetic pull due to an unsymmetrical magnetic field in a badly designed machine, or in a machine with a disc armature on which the thrust rings are worn. The wearing of the thrust rings may allow the armature to move sufficiently to change an otherwise symmetrical field into an unsymmetrical one, and thus throw a considerable pressure on the thrust bearings, on account of the tension of the lines of force.

The C^2R losses in armature and fields can be quite accurately determined by measuring the hot resistances and the currents flowing. The loss due to foucault currents in the armature conductors cannot be measured directly, but it enters into the general determination of foucault current losses. The loss due to friction can be measured by some form of dynamometer, the machine being run without current in either the field or armature. The friction loss thus measured evidently may not represent the true working friction, but it usually is a fair approximation in well-designed machines. There are two fairly good methods which may be used in measuring and separating the losses in an armature due to foucault currents and hysteresis. Neither of the methods takes account of the losses due to foucault currents in the pole pieces, as these are due to the effect of the armature current, and neither of the methods admits of using sufficient current in the armature to appreciably affect the pole pieces.

First method. Drive the machine to be tested without load, and at normal speed V. By a sensitive dynamometer measure the power $W(=FV)$ required to overcome friction. From an external source excite the fields to their normal magnetization under full load. By the dynamometer measure the power W required to overcome the friction loss FV, the hysteresis loss HV, and the foucault current loss ZV^2. Again, measure the power W' required to overcome the losses at the speed V', with the field magnetization remaining constant. Then

$$W=FV+HV+ZV^2,$$

$$W'=FV'+HV'+ZV'^2.$$

$$\therefore\ Z=\frac{WV'-W'V}{V'V^2-VV'^2}$$

and
$$ZV^2=\frac{(WV'-W'V)V}{V'V-V'^2};$$

also
$$F+H=\frac{W'V^2-WV'^2}{V'V^2-VV'^2}$$

and
$$FV+HV=\frac{W'V^2-WV'^2}{V'V-V'^2}$$

and
$$HV=(FV+HV)-W_f.$$

Second method. Drive the machine as a motor without load at normal speed V, and at normal excitation. Measure the current C, which flows through the armature, and the pressure E at the terminals. Then, neglecting the C^2R loss in the armature, $CE=$ the sum of losses due to friction, foucault currents, and hysteresis at speed V. Keeping the excitation constant, change the pressure at the armature terminals and measure the current C', pressure E', and the resulting speed V'. $C'E'$ represents the sum of the losses at speed V'.

Then
$$CE=FV+HV+ZV^2,$$

$$CE'=FV'+HV'+ZV'^2.$$

$$\therefore\ Z=\frac{CEV'-C'E'V}{V'V^2-VV'^2}$$

and
$$ZV^2=\frac{(CEV'-C'E'V)V}{V'V-V'^2};$$

also
$$F+H=\frac{C'E'V^2-CEV'^2}{V'V^2-VV'^2}$$

and $$FV+HV=\frac{C'E'V^2-CEV'^2}{V'V-V'^2}.$$

Again, neglecting the armature resistance,

$$V=\frac{60\times 10^8\times E}{N_a\times S}=pE \text{ and } V'=pE'.$$

$$\therefore\ ZV^2=\frac{(C-C')}{E-E'}E^2$$

and $$FV+HV=\frac{(C'E-CE')}{E-E'}E.$$

The values of ZV^2, $FV+HV$, or HV, determined by these methods, differ somewhat from the values when the machine runs under load, on account of the influence of the armature current. ZV^2 is also likely to be greater in a motor than in a dynamo. In most cases, however, the measurements thus made are likely to approximate closely to the actual values when the dynamo is under load.

Skill, judgment, and carefully standardized instruments are necessary for the *experimental determination* of the commercial efficiency of dynamos, so that the result is known to be accurate within 1 per cent of its value. In efficiency tests, the most serious errors are likely to be introduced in the measurement of the mechanical power which is absorbed by the dynamo, as power measurements by dynamometers are subject to considerable uncertainty. The most convenient dynamometer, for use with dynamos or motors, probably is a properly arranged Bracket cradle or floating dynamometer, but even this is likely to give contradictory and

erroneous results. In the case of a motor, a friction brake can be made to give fairly good results. Check measurements, to guard against errors, should always be carefully made. Unfortunately no check on the dynamometer can be made except by using a transmission dynamometer to measure the belt pull, and transmission dynamometers are notably unreliable. On the other hand, electrical measurements can be made to any desirable degree of refinement, and any existing errors can be readily checked and compensated. It is therefore desirable to reduce efficiency tests to measurements of electrical units only, when considerable accuracy is desired. This can be done by the use of three machines, arranged as shown in Fig. 126. *D* is the dynamo to be tested. It is connected by wires of known resistance to a similar machine, *M*, which is run as a motor. The shafts of the two machines are connected by a coupling or a belt. If sufficient electrical energy to compensate for the losses in *D*, *M*, and the connections, be furnished from another dynamo, *L*, the system can be made to run at any desired load on *D*, merely by varying the strength of the field of *M*. By the amperemeter *A*, and the voltmeter *V*, the output of *D* can be measured ($=CE$). By the amperemeter *A'*, and the voltmeter *V'*, the power sup-

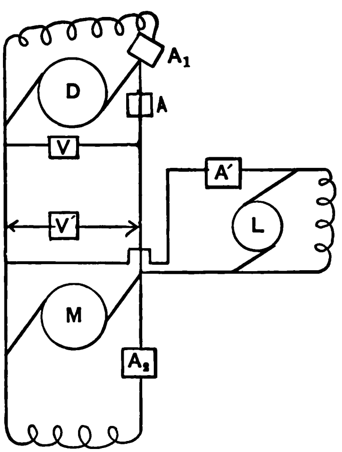


Fig. 126.

plied to make up the losses can be measured ($=C'E'$). If R_L be the resistance of the connections, C^2R_L is the loss in them, and $C'E'-C^2R_L$ is equal to the total loss in the two machines. Assuming that this is equally divided between the machines, the commercial efficiency of the dynamo is, $\eta=\frac{CE}{CE+\frac{1}{2}(C'E'-C^2R_L)}$. The truth of the assumption that the losses are equal in the two machines can be checked by measuring the armature resistances, field currents (by amperemeters A_1 and A_2), and the value of $FV+HV+ZV^2$ for each machine. If the losses are not closely equal, a proper correction can be applied to the efficiency as determined by the test. The third dynamo can be replaced by a storage battery, if desirable, or mechanical instead of electrical energy may be furnished the system, to compensate for the losses. Since in the latter case the power measured by a dynamometer is a small percentage of the total energy of the machines, an error of several per cent in its measurement only slightly affects the result. The amperemeters and voltmeters, used in the electrical measurements, must be standard measuring instruments of known accuracy. Probably the most accurate and satisfactory series of efficiency tests, of which official reports are obtainable, were those made after the Franklin Institute Electrical Exhibition, in 1884. The power measurements were made with a Tatham dynamometer specially constructed, and the electrical measurements were made with sensitive galvanometers, accurately standardized by comparison with a silver voltameter and tangent galvanometer. (Refer to *Report Electrical Exhibition*, 1884.)

For ordinary commercial purposes, power measurements made with a Bracket cradle suffice, and electrical measurements can be made with properly selected and standardized Weston, Wirt, or similar portable instruments. In this way the error is greater than when the more reliable methods are used, but it can probably be reduced to within two per cent.

If the loss in the fields, and HV, FV, and ZV^2 be considered constant in the case of a shunt dynamo under varying loads, it can be shown that if the external characteristic has a small slope the maximum commercial efficiency occurs at the output when the variable armature loss equals the sum of the other losses. For, if Ce be the output, C^2R_a the armature loss, and L the other losses, then

$$\eta=\frac{Ce}{Ce+L+C^2R_a}=\frac{1}{1+\frac{L+C^2R_a}{Ce}}.$$

To give a maximum efficiency, $\frac{L+C^2R_a}{Ce}$ must evidently be a minimum. Solving for the minimum in the usual manner there results

$$R_a-\frac{L}{C^2}=0, \text{ or } C^2R_a=L.$$

In the example of a 25 K.W. machine already discussed, the field loss may be considered as constant at 675 watts, and the other constant losses as about 1000 watts. For maximum commercial efficiency, C^2R_a must be 1675 watts or $C=350$ amperes. At normal load,

$$\eta=kf=96.3\times 95.5=92.0 \text{ per cent.}$$

At 350 amperes,

$$\eta = k'f' = 97.9 \times 94.9 = 92.9 \text{ per cent.}$$

The gain in efficiency, caused by increasing the output from 200 amperes to 350 amperes, is thus shown to be small. Moreover, a portion of the losses which were assumed to be constant would undoubtedly increase considerably, thus actually causing the efficiency to decrease. It is therefore useless to make any sacrifices in order to gain increased armature currents with a view to raising the efficiencies. The difficulty of avoiding sparking, and of keeping the armature cool, without sacrificing good construction, much more than compensate the advantages derived from so large a current in the armature. It is to be noticed that the maximum commercial efficiency requires a larger load than the maximum electrical efficiency, hence it is well to design a shunt machine with $R_a > \frac{R^2}{R_f}$ when this can be done without causing excessive heating in the armature.

In series and compound dynamos the conditions for maximum commercial efficiency are more obscure, but judgment can be guided by the conditions demanded in shunt machines.

Variation of Efficiency, Weight, and Cost, with Output.

The electrical efficiency that can be economically attained in any dynamo depends upon its capacity, as is shown by an inspection of the tables of allowable losses in armature and field windings previously presented (see pages 108 and 139). The tables also show

that the percentage loss in the field windings decreases more rapidly than that in the armature windings, as the machine is increased in size. The efficiency of conversion also increases with an increase in the capacity of dynamo, but not as rapidly as does the electrical efficiency. In either case, the rate of change in the value of the efficiencies, as the capacity is increased, is

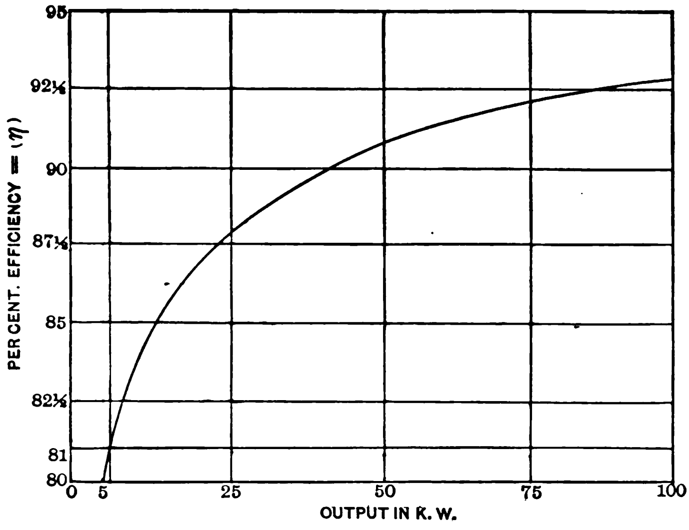


Fig. 127.

greater in smaller machines than it is in large ones. Since the commercial efficiency η, is equal to the product fk, it must also increase with the dimensions and capacity of dynamos. By plotting the values of η that may be expected in dynamos of various capacities and good design, when operated under full load, the accompanying curve (Fig. 127), is found. The equation of

this is of the form $\eta = 100 - \frac{q \times \text{K.W.}}{\sqrt[3]{W}}$, where q is a quantity proportional to $\frac{1}{2\pi r V B}$, and K.W. and W are respectively the normal output in kilowatts and the weight in pounds. If in any particular type of dynamo the surface speed of the armature ($2\pi r V$) and the value of B are assumed to be constant in all sizes of the type, the value of q is constant. Then, using the average value for W as determined by a succeeding formula, it is found that the value of q may be considered to be approximately 180 for dynamos of capacities between 5 K.W. and 100 K.W., and the equation of the average curve of efficiencies becomes $\eta = 100 - \frac{180 \text{ K.W.}}{\sqrt[3]{W}}$. This equation can be taken to fairly represent the average efficiencies which have been attained in good practice with dynamos of usual types and design, but too much reliance must not be put upon its indications, as it is what may be called a semi-empirical formula; that is, it is based to some degree on theoretical grounds, but practical modifications have entered largely into it. A sufficient amount of data has not been examined to assure its fairly general application, and in any event it is scarcely to be expected that any formula can do more than roughly approximate to a correct expression of the results of the many variable combinations which may control the commercial efficiency of a dynamo. The efficiencies of some of the most modern dynamos considerably surpass the indications of the formula.

In view of the increase in efficiency as the size of dynamos is increased, and of the smaller proportional

space occupied by insulation on the windings of large dynamos, the output of any type of dynamos may be expected to increase at a more rapid rate than the weight, as the size of the machine is increased. The ratio of output to weight may also be expected to change most rapidly in machines of comparatively small size, and to change quite slowly in machines of over 25 or 30 K.W. capacity, since the efficiency of a properly designed 25 K.W. machine has already been shown to be little less than that attainable in the largest two-pole machines. Various ratios between output and weight have been suggested, many of these being based simply on *a priori* reasoning. The early propositions of Deprez and S. P. Thompson which made the output of machines having similar form, but different weights, proportional to the 1.7 power of the weights (the fifth power of the linear dimensions), had as a basis certain *a priori* assumptions which experience has proved to be erroneous. Some later deductions based on conditions more nearly approximating those of practice, make the output vary directly as the weight, while others make it vary as the 1.17 power, and still others make it vary as the 1.33 power. These are equivalent, respectively, to the cube, $3\frac{1}{2}$ power, and fourth power of the linear dimensions. There is in truth a wide range of variation in the relation between output and weight, as shown by an examination of a large number of machines of various well-known manufacturers, and therefore any reasoning based on the assumption of a constant relation between the linear dimensions of different sizes, even of the same type of machine, must fail on account of error in

the assumption. In order to determine the average relation between output and weight, an output-weight curve must be plotted for many different types of dynamos which have capacities between the limits which the investigation covers, a safe temperature limit, and an armature velocity near 3000 feet per minute. For capacities from 5 K.W. to 25 K.W., this curve proves to be concave towards the axis of X, and its equation is K.W. $=\frac{W^{1.16}}{500}$, where K.W. represents the output in kilowatts, and W the weight in pounds. For capacities greater than 25 K.W., the locus proves to be a practically straight line represented by the equation K.W. $=\frac{W}{170}+5$. By plotting the points for a great number of machines, regardless of their limiting temperatures when under full load, or of the periphery speeds of their armatures, the limiting relations between output and weight likely to exist in commercial machines can be defined. Thus, a line passing along the lower edge of the zone of points, and representing machines with a high temperature limit, or armature velocity — or both — has the equations

$$\text{K.W.}=\frac{W^{1.2}}{500} \text{ and } \text{K.W.}=\frac{W}{130}+6$$

for capacities, respectively, below and above 25 K.W., within the minimum and maximum limits of 5 K.W. and 100 K.W. Some dynamos of makers with excellent reputations fall near this line, but in some of these cases the same makers build more expensive machines of equal capacity which fall nearer the average line.

The higher limit of the zone of points is represented fairly well by a single straight line with the equation $K.W. = \frac{W}{220}$. The machines represented by points near this line can be regarded either as special slow speed dynamos designed to be used directly connected to engines, or as machines which, for some reason, have been given an excessive weight, and which would not be of economical design for general service on account of their high cost.

It is therefore safe to say that machines of the best types, designed for ordinary service, will increase in capacity up to about 25 K.W. output, as the 1.16 power of their weight. That the rate of increase in output cannot safely exceed the 1.2 power of the weight, and that for general purposes machines with an output which increases at a rate less than the 1.12 power of the weight must be looked upon as unnecessarily heavy. For capacities greater than 25 K.W., and at least up to 100 K.W. capacity, the output of two pole machines can be considered to vary in nearly direct proportion with the weight.

It is usual for the cost of machinery of any nature to become smaller per pound of material as the total weight is increased, unless some special conditions are met, and the general rule may be applied to most types of dynamos. In general it may be said, that the cost of a dynamo of greater capacity than 5 K.W. is about in proportion to the .8 power of its weight (cost $\propto W^{.8}$), but in some types the cost is more nearly in direct proportion to the weight. Taking the .8 power as a fairly general rule for the total cost of machine, the cost per

pound of weight must vary inversely as the fifth root of the weight $\left(\frac{\text{cost}}{W} \propto \frac{1}{W^{\frac{1}{5}}}\right)$. A more desirable comparison of the cost of machines of different sizes is the cost per unit of output. This can be directly deduced from the relations between total cost and weight, and between output and weight, which have already been given.

Assuming for machines of capacity between 5 and 25 K.W., the average ratio of output to weight, there results $\frac{\text{cost}}{\text{K.W.}} \propto \frac{W^{.8}}{W^{1.16}} \propto \frac{1}{W^{.36}}$, which is nearly equal to $\frac{\text{cost}}{\text{K.W.}} \propto \frac{1}{W^{\frac{1}{3}}}$, that is, the cost per unit of output varies inversely as the linear dimensions.

For machines of capacity greater than 25 K.W., it has already been shown that output varies in nearly direct proportion to weight, hence $\frac{\text{cost}}{\text{K.W.}} \propto \frac{1}{W^{\frac{1}{5}}}$, which shows that in dynamos larger than 25 K.W. the gain in cost per K.W., by increasing the capacity of individual machines in a plant, is by no means as great as is the case with smaller machines. The maximum cost per K.W., which is likely to be found in any dynamo greater than 5 K.W., may be set down as $\frac{\text{cost}}{\text{K.W.}} \propto \frac{1}{W^{\frac{1}{3}}}$. Another relation which may sometimes be useful for comparison is that existing between the total cost and the total capacity of the dynamo. For dynamos of capacities between 5 and 25 K.W. this can in general be approximately written, cost $\propto \overline{\text{K.W.}}^{.7}$.

As already intimated, so many diverse conditions exist in the practical design, manufacture and operation of dynamos, that a satisfactory set of premises upon

which to base theoretical deductions of the results here discussed, cannot be presented. The deductions given here are derived purely from an examination and comparison of the weights, outputs, and cost of marketable dynamos, and they are quite similar, in some respects, to those presented by Hopkinson and Kapp as based to some degree upon *a priori* reasoning, but qualified by their experience as designers of dynamos. Hopkinson puts the output as directly proportional to the cube of the linear dimensions; while Kapp puts it as proportional to the 3½ power of the linear dimensions. Kapp also states that the total cost of dynamos when of less capacity than about 35 K.W., varies as the 2½ power of the linear dimensions, and that in larger machines it varies more nearly as the cube of the dimensions, while the cost per unit of output varies inversely with the linear dimensions.

In making use of these data, it must be remembered that varying requirements of service demanded from machines of the same type, will often alter the relative proportions. Thus, when a dynamo is wound for a given output at one hundred volts pressure, the space occupied by insulation on the armature will be considerably less than when the machine is wound for ten or twelve hundred volts. This condition may make it impossible to wind the same frame for an equal output at widely differing pressures, and the cost and weight per unit of output for the same frame may be quite different for different windings. It may be safely said, however, that up to the limit of from 1000 to 1200 volts, any extra space required for insulation at the higher pressures, can usually be provided for by reducing the num-

ber of conductors on the armature and raising the speed proportionally. In this case, no change occurs in the relation of cost and weight to output.

In dynamos of capacities less than 5 K.W., the increased proportional depth of the necessary mechanical clearance, and the larger proportion of the winding space occupied by insulation, causes the curve showing the relation between commercial efficiency and capacity to fall very rapidly, and finally to cut the X axis at a short distance to the right of the origin. This shows that all the electrical energy which the armature of a dynamo of certain dimensions is capable of developing, is required to excite the fields and overcome the C^2R loss in the armature. Dynamos of a still smaller size are unable to excite themselves at all. The limit of power to excite is apparently at a capacity of about 75 watts, except in machines built with every precaution to reduce the C^2R losses and the reluctance of the magnetic circuit. On the other hand, the smallest machines may be run as motors, though at a low efficiency, for sufficient energy can be supplied to properly magnetize the fields and make up the various losses, without regard to the electrical output of which the armature is capable. In well-built machines, of capacities greater than 5 K.W., the curves representing the machines operated as dynamos or motors, are practically coincident. In the case of smaller machines, the curves must diverge, since the motor curve passes through the origin, and therefore the efficiency of a small machine is likely to be greater when used as a motor, provided no disturbing influences enter.

The accompanying figures, 128 and 129, represent some useful relations between the external characteristic, load, and efficiency of a dynamo. Figure 128 represents the load and **Economic Curve** for a shunt dynamo, and Fig. 129 shows the same for a series dynamo. Each machine has a normal output of about 6000 watts. In

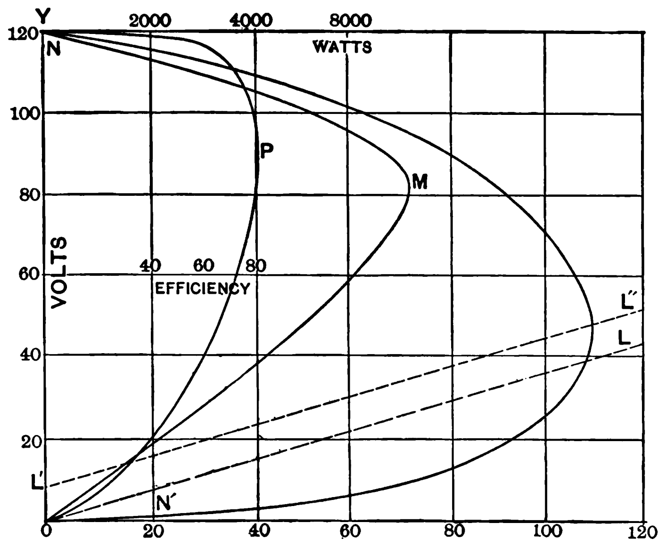


Fig. 128.

each case the curve NN' represents the experimentally determined external characteristic and OL the loss line. The ordinates of the line $L'L''$ are equal to the corresponding ordinates of the loss line plus the measured value of the loss due to hysteresis, foucault currents, and friction, which is assumed to be constant. The line $L'L''$ is therefore parallel to the line OL. The curve

OMY represents the total output of each machine when working at each point of its external characteristic, and the economic curve *OPY* represents the commercial efficiencies at the various loads. The small change in the efficiencies as the loads decrease from full load to ¼ load is to be remarked in each case. Since the

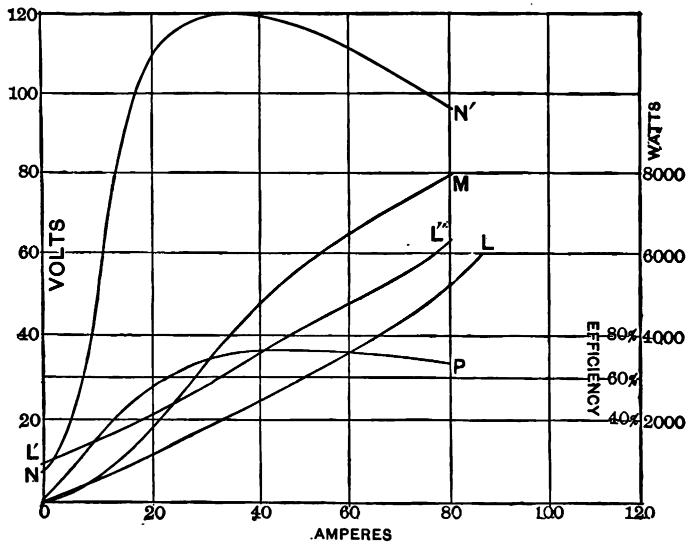


Fig. 129.

economic curves of these figures were not determined directly by experiment, but were derived by computation from experimental characteristics, and moreover, dynamos when in use are commonly regulated for constant pressure, by changing the field losses, it is useful to compare their form with that of similar curves plotted from the records of several tests. Such are the following

curves (Fig. 130), the first four of which are the results of tests made on shunt dynamos at full, ¾, ½, and ¼ load, and the fifth, the result of tests on a series machine. All the machines were run at an approximately constant pressure. The differing slopes in the curves thus

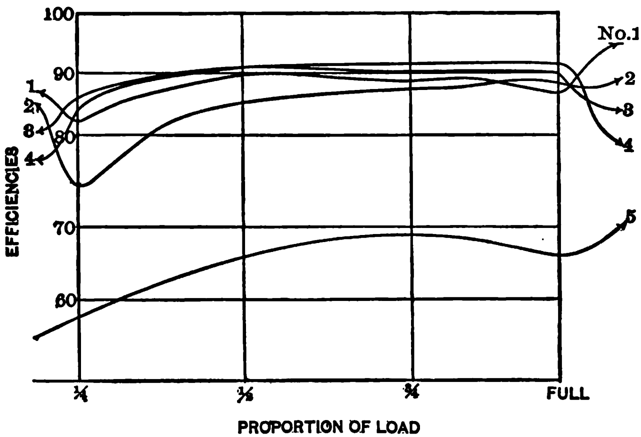


Fig. 130.

No. 1. Capacity, 9.6 K.W.
No. 2. " 10 K.W.
No. 3. " 20 K.W.
No. 4. Capacity, 50 K.W.
No. 5. " 15 K.W.

plotted are evidently shown, by what has preceded, to be due to differences in the external characteristics and loss lines of the dynamos, and the similiarity in the form of the directly and the indirectly determined curves is quite close.

CHAPTER IX.

MULTIPOLAR DYNAMOS.

It has been already shown (Chap. VIII.) that the output per unit of weight of large two-pole dynamos does not materially increase with increasing size, and some more economical form for large dynamos is therefore sought. Any change in the form of the machines should be one which decreases the total cost of the machines per pound of weight. The desirability of a change in the form of large machines is made more imperative by the increased difficulty in suppressing heating and sparking in two-pole dynamos of capacities greater than 100 kilowatts. In order that what may be called the **Weight Efficiency** (output per pound of weight) may be increased without increasing the periphery velocity of the armature, or dangerously increasing the temperature limit, it is necessary to decrease the reluctance of the magnetic circuit. In order to reduce the tendency of large machines to sparking, the ratio which the armature cross ampere turns bear to the field ampere turns must be made as small as possible. Only one practicable method for reducing the reluctance of the magnetic circuit presents itself; that is, to reduce the ratio which the length of the air gap bears to the area of its cross-section. To do this requires an increase in the area of the pole faces. As mechanical considerations limit the

length of dynamo armatures, it becomes necessary in large machines to increase the armature diameters, in order to gain the desired increase in the area of the air gap. The great mass of iron in such armatures is found to heat excessively when the ordinary methods of construction are used, and much difficulty is experienced from the effect of armature reactions on account of the great width of the pole pieces. It is therefore advantageous to divide the magnetic circuit, making dynamos with two or more pairs of poles. These are called **Multipolar** dynamos to distinguish them from the two-pole or **Bipolar** form.

Increasing the diameter of an armature allows a greater circumference in which to wind conductors, and therefore the depth of winding may be proportionally decreased. Armature windings for multipolar machines follow the same fundamental laws as for two-pole machines. Simple Gramme rings may be used, as shown in Fig. 131, in which case there will be as many neutral and commutating planes as there are pairs of poles. A pair of brushes may touch the commutator upon each of the intersections of the commutating planes with the commutator. Since the poles are alternately N and S, the brushes will be alternately positive and negative. When one-half of the number of poles (*i.e.* the number of pairs of poles) is an even number, opposite brushes are of the same polarity, as in Fig. 131, which shows a four-pole dynamo. When one-half the number of poles is not an even number, opposite brushes are of opposite polarity, as in Fig. 132.

When the number of poles is double an odd number, a

plain Siemens drum winding may be used, but it cannot be used when the number of poles is double an even number, since the two halves of any coil would then pass simultaneously before poles of like sign, and the effective pressure induced would be zero. A **Chord Wound** drum armature may be used with any number of pairs of poles, provided each coil be wound to cover an

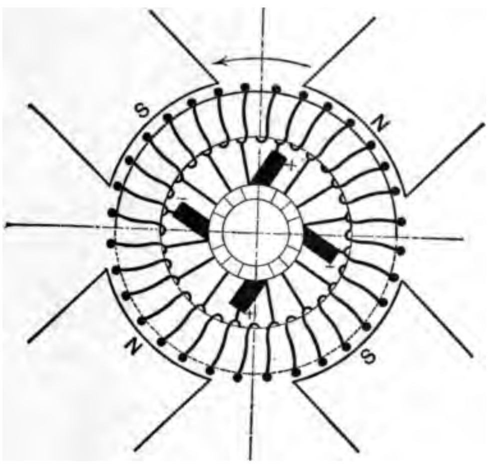


Fig. 131.

angular arc on the armature surface equal to nearly the angular pitch of the poles. When drum windings are used, the number of brushes required is the same as in the case of ring windings; that is, the current is divided in the armature between as many paths as there are poles. It is usual to connect the brushes of like polarity together, so that the paths are actually placed in parallel, and the output of the machine may be used as one unit. Unless the armature is in excel-

lent electric and magnetic balance, and all the magnetic circuits of the field have an equal effect on the armature, excessive heating and sparking is bound to result from this arrangement. If the sector of the armature between one set of brushes does not generate exactly the same

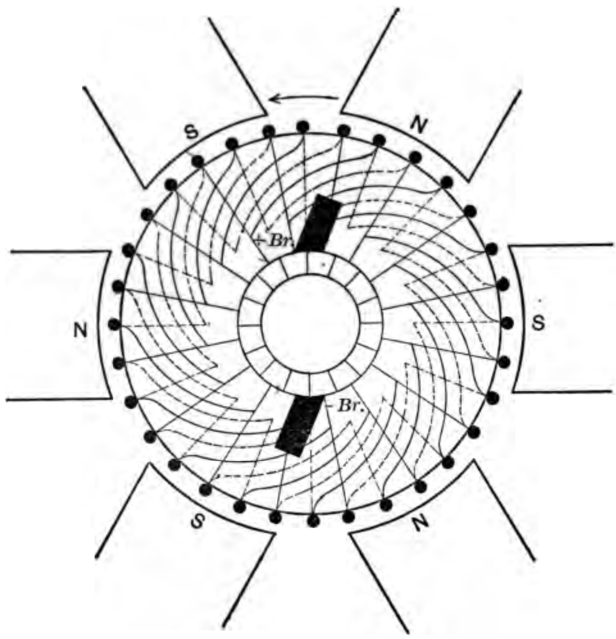


Fig. 132.

pressure as all the other sectors, an interference is at once set up in the armature circuits. Since the resistance of the current paths in the armatures of large multipolar dynamos is very low, these interferences are likely to reach a destructive magnitude, if great care is not observed in designing the machines. In some small

multipolar machines the fields are so arranged that the pole pieces may be adjusted until all the sectors show exactly the same difference of pressure by a volt-meter test (example, Perrett motors). The effect of slight irregularities in electric balance is shown by the operation of one type of large multipolar dynamos in which a single turn Gramme wound armature is used. The armature core is mounted on a spider, and the slightly increased length of the inside conductors where they pass around the spokes of the spider throws the armature sufficiently out of electric balance to cause sparking, accompanied by considerable burning, at the commutator bars which are connected to the longer coils. Wear of the bearings may cause the armature to take an eccentric position between the pole pieces, and thus throw it out of magnetic balance. The effect of this is to cause unequal pressures in the armature sectors, so that the load is unevenly divided, with sparking and heating as a result. The pressures in the sectors may become so unequal that one may even be overpowered, and the current be reversed in it. This condition may also be caused by constructive defects in one or more of the magnetic circuits. The connection of the armature circuits in parallel makes their action virtually similiar to that of two or more separate machines connected in parallel. Each must generate the standard pressure without peradventure, or disaster is likely to result.

To overcome the defects of the **Multiple Path Winding,** great precautions must be taken in designing the bearings and the magnetic circuits of multipolar dyna-

mos. Trouble from any ordinary wear of the bearings may be avoided by making a considerable part of the reluctance of the magnetic circuit in the iron of the field magnets. The induction in the field magnets of multipolar machines may often, therefore, be advantageously greater than in the field magnets of two-pole dynamos. The use of **Iron Clad** or **Embedded** armatures is also of assistance in this point, but the difficulties of avoiding sparking with such armatures leads to the use of very high inductions in the clearance space. In some cases the mechanical clearance is made quite large to reduce the effect of the armature cross-turns (see Chap. VI.), and the induction in the air gap may reach 10,000, or even more, lines to the square centimeter.

The troubles due to differences in the magnetic circuits may be entirely eliminated by winding the armature so that the current is divided between only two paths, exactly as in two-pole machines. This may be called a **Series** or **Two Path Winding.** When the series path winding is used the wire of each coil must cross the face of the core as many times as there are field poles, the turns being spaced at a distance equal to nearly the pitch of the poles. Thus, in a four-pole machine, coil number one may start at commutator bar number one, pass across the face of the drum to the back end, across nearly a quadrant of the core, forward across the face of the drum, across nearly a quadrant, back across the drum, across nearly a quadrant, forward across the drum, and finally across nearly a quadrant to commutator bar number two. The winding passes across the ends of the core always in the same direction, so that it makes a zigzag

course completely around the drum. The connections of series path armatures and the direction of winding follow the same laws as those of two-pole armatures.

Gramme windings may be arranged in series path by electrically connecting the proper coils in series. Series path armatures will operate satisfactorily, regardless of inequalities in the strength of the magnetic circuits. Unless specially arranged, series path windings require only two sets of brushes, and the brushes are 180 degrees apart in machines having an odd number of pairs of poles, and at an angular distance apart equal to the pitch of the poles in machines having an even number of pairs of poles. It is not uncommon to arrange the commutators with twice as many bars as there are coils in the armatures, in which case the extra bars are properly cross connected to the active bars so that four sets of brushes may be used, in order to give a greater current carrying capacity. A somewhat analogous arrangement of cross connections may be used with multiple path windings to reduce the number of brushes to two sets. Here bars of like potential are connected together; thus in a four-pole machine, with a multiple path armature, opposite brushes are of like polarity, and therefore if opposite commutator bars be connected together only two sets of brushes are required for operation. The mechanical arrangement of the cross connection is effected in a great variety of ways which it is not necessary to describe.

The number of conductors to be placed on the armature of a multipolar machine is determined exactly as in the case of a two-pole armature. If the notation of Chap. IV.

be retained, but S' be made to represent the number of conductors required to develop the electric pressure E, N the number of lines of force passing into the armature from each pole, and S the total number of conductors on the armature; then for multiple path windings, if p represent the number of pairs of poles,

$$S' = \frac{10^8 \times 60 \times E}{p \times N \times V}.$$

Since p sectors of the armature must develop the pressure E,

$$S_m = pS' = \frac{10^8 \times 60 \times E}{N \times V}.$$

In series path windings the pressure E is developed by half the armature conductors in series, and each coil cuts as many lines in one revolution as a coil upon a multiple path armature (that is, $2pN$ lines). Therefore

$$S_s = S' = \frac{10^8 \times 60 \times E}{p \times N \times V}.$$

These equations show that the general formula for determining the number of conductors on any armature is

$$S = \frac{10^8 \times 60 \times E}{p \times N \times V} q,$$

it being remembered that p is equal to one-half the number of paths through which the current in the armature divides, and N is the number of lines of force due to each pole (*i.e.* the number of lines in each magnetic

circuit); and q being equal to unity and to p, respectively, in series and multiple path windings. Since the current divides between two paths in the series path winding and between $2p$ paths in the multiple path winding, the conductors of a series path winding must have p times the cross-section of the conductors of a multiple path armature which is wound for the same output, and the total weight of copper on the two armatures is practically the same. In drum armatures of moderate size the series path winding is in general somewhat less expensive to build than the multiple path armature, since the labor in winding an armature, with x turns of wire having a cross-section A, is usually less than in winding one with px turns of area $\frac{A}{p}$, and the insulation of the finer wire makes its cost slightly greater. A little consideration makes it evident that the total number of conductors on a series path armature must equal $2px \pm 2$, where x is any number. When the $+$ sign is used, the pitch of the end connections is a little less than the pitch of the poles, and when the negative sign is used it is a little greater.

Experience shows that properly designed four-pole dynamos are, in general, unquestionably more economical in construction than two-pole machines for capacities greater than 50 kilowatts; and also in general, that machines of less than 25 kilowatts capacity can be more economically built of the bipolar type. Machines of very great capacity cost less when constructed with more than four poles, the best number of poles depending on the capacity of the machine and the service required of it.

It is not unusual to sacrifice some of the weight efficiency of multipolar dynamos for the purpose of reducing the rotative speed of the armatures. In this case it is usual to make the armatures particularly large in diameter, in order that the low rotative speed may be gained without too great a reduction of periphery velocity. To fully utilize the large circumference of such armatures, the number of poles is usually multiplied, but the difficulties due to armature reactions seem to be increased with the number of poles. This is apparently due in considerable part to the flat form which is necessary in pole pieces which are on the outside of large armatures, and which therefore cannot be readily designed to make the field stable. In some cases the field can be stiffened by constructing the pole corners with thin extensions which become thoroughly saturated by magnetic leakage lines. This however sacrifices some magnetizing power. Apparently the pole pieces are more readily made of a satisfactory form, where large armatures are used, when the armature revolves outside of the field magnets. (Example, large Siemens and Halske dynamos.) In general it may be said that great reductions of rotative speed can only be obtained either by considerable sacrifice in weight efficiency, or by sacrificing sparkless operation. The first, when carried to an extreme, makes too expensive a machine, and the latter causes increased repairs and depreciation of the machines when in operation. By following a mean position, excellent multipolar machines of large capacity are now built with such low rotative speeds that they may be satisfactorily connected direct to any type of

steam engine. (Examples, Westinghouse Kodak and Corliss direct connected sets, General Electric direct connected sets, Siemens and Halske steam dynamos, and those of many other domestic and foreign makers.)

In multipolar dynamos the leakage coefficient varies from 1.20 to 2.00, depending upon the number of poles, their closeness together, and the reluctance through the air space and armature. The former value is only met with in machines with embedded conductors and a shallow air space. The latter value belongs to machines with leakage paths of abnormally low reluctance and a deep air space, a type which is now not generally built.

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